INVESTIGATION OF DETERMINISTIC CHAOS BY V-SCOPE MOTION TRACKING SYSTEM

PÉTER VANKÓ
Dept. of Experimental Physics, Technical University of Budapest, Hungary
E-mail: vanko@goliat.eik.bme.hu

and

LÁSZLÓ GRÁNÁSY
Research Institute for Solid State Physics and Optics, Budapest, Hungary
E-mail: grana@power.szfki.kfki.hu

Abstract

First-year physics students of the Technical University of Budapest carry out a wide range of measurements in the Basic Level Physics Teaching Laboratory. One of the most exciting experiments is the investigation of a chaotic double pendulum by V-scope, a powerful 3D motion tracking system. In this workshop, after a brief introduction to the V-scope apparatus, measurements on chaotic motion will be performed. The collected data are going to be processed by V-scope for Windows and Matlab 5.1 software.

KEYWORDS: chaotic motion, double pendulum, motion tracking, V-scope

Introduction

The Basic Level Physics Teaching Laboratory of the Technical University of Budapest offers a wide range of measurements for the first and second year physics students. The experiments are selected so that besides observing fundamental physical phenomena, the students gain experience in using a variety of measuring devices ranging from simple to some rather sophisticated ones. Further aims are that they become familiar with fundamentals of error calculus and data processing.

In this workshop we present one of our most spectacular experiments: the investigation of a chaotic double pendulum by V-scope, a powerful 3D motion tracking system. After a brief introduction to the equipment, various mechanical phenomena will be investigated. The respective data will be processed by *Vscope for Windows* and *Matlab 5.1*.

The Chaotic Double Pendulum

The double pendulum is built of two pendula attached to each other via a common axis (*Fig. 1*). It is one of the simplest mechanical systems that exhibit chaotic behaviour. Chaotic systems are known to be sensitive to the initial conditions (i.e., slight initial differences quickly evolve into apparently different states) and exhibit irregular, unpredictable behaviour.

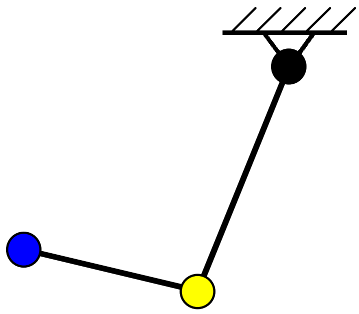

Fig. 1

The motion of the double pendulum is described by the appropriate Euler-Lagrange differential equations [1], which can be solved numerically for any particular choice of the parameter values and initial conditions. The solutions display a fairly complex behaviour akin to that seen in experiments.

While numerical simulations are available on the net in abundance and are used in physics teaching widely, we present here *real* experiments. Although the surprisingly complex behaviour and the evidently different motions that follow from virtually identical initial conditions are inspiring on their own; it is far more impressive when the evolution of the system can be quantified via tracking with the V-scope.

The V-scope Motion Tracking System

The V-scope system [2] uses a unique tracking technology that allows full three-dimensional tracking of up to four bodies within a space of few meters, with a resolution of a fraction of a millimetre. The parts of the V-scope system that execute the tracking process are the *Buttons*, the *Towers*, the *V-scope Microcomputer* (*Fig. 2*) and the *V-scope for Windows Software*.

Fig. 2

The *Button* is the smallest yet most sophisticated part of the system. It is a battery-operated infrared-ultrasonic transponder attached to the moving bodies to be tracked.

Each Button has a different triggering code, which allows selective scanning. On-off switching of the Button's power is done by remote signals, extending the battery life to years of maintenance-free operation. Three Buttons, distinguishable by colour, are supplied with the system. Special self-adhesive, snap-on mounts are provided, to be attached to the desired bodies or points to be tracked. A Button is firmly attached or easily detached from such a mount with a single click.

The *Tower* is a stationary infrared-ultrasonic transceiver. It transmits a coded infrared signal to trigger selectively a desired Button, and receives then the ultrasonic response signal. A thermistor that measures the ambient temperature is located in the base of each Tower. A special Tower Mount is provided, to allow automatic calibration of the system.

The *V-scope Microcomputer* controls the operation of the Towers, receives and processes signals, and calculates position data. Results are transmitted to the PC via an RS-232 interface.

The *V-scope for Windows Software* actuates the PC to receive and record positioning data, processes, stores and displays these data, and provides a user-friendly interface. The motion related quantities can be viewed on-line or in replay. The V-scope tracks the 3-D position versus time (x,y,z vs. t) for the active buttons. A selection of motion-related quantities are automatically determined (on-line). These include the velocity, acceleration, momentum and their spatial components - if masses are defined, amplitude, period and frequency - if the system identifies a periodical motion and any user defined function like kinetic and potential energy, relative position, angular or relative velocity, etc.

The screen arrangement used during the experiments of the workshop is shown in (*Fig. 3*): *XY graph*: XY plane with trace of both (blue and yellow) buttons, *table*: X and Y position of both buttons versus time, *Xrel Yrel graph*: the relative position of the blue button to the yellow button (without trace).

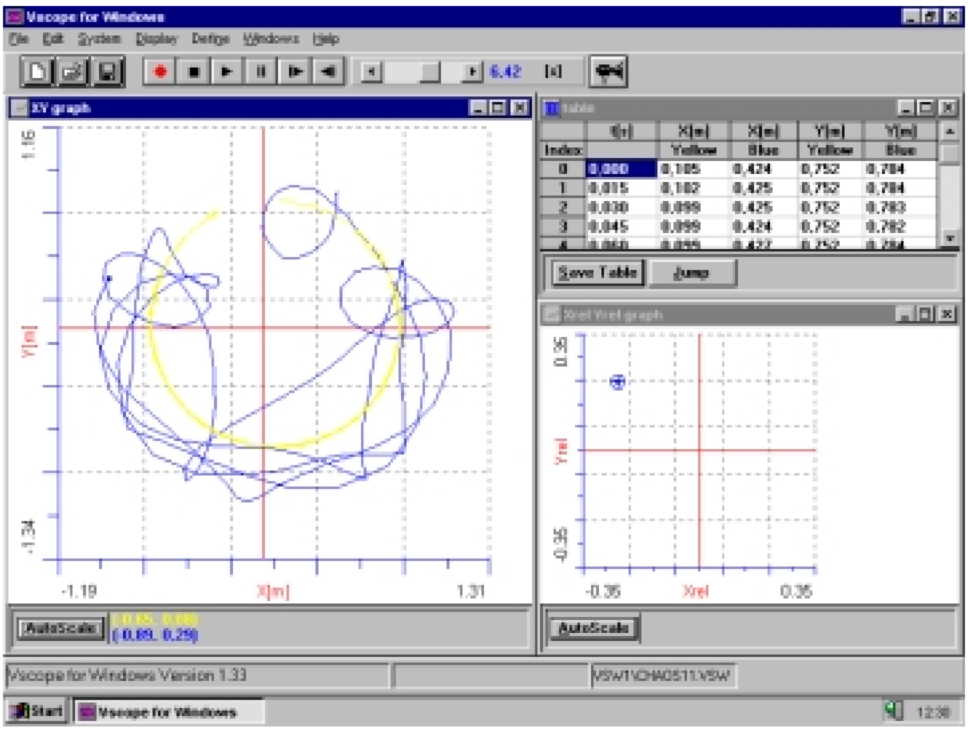

Fig. 3

Measurements in the Workshop

Before investigating the behaviour of the chaotic double pendulum, it is instructive to demonstrate the abilities of V-scope for simpler measurements.

Motion Challenge

In this experiment, a button has to be moved so, that the XY and XZ graphs shown in *Fig. 4* are recovered. It is certainly not that easy, as it seems!

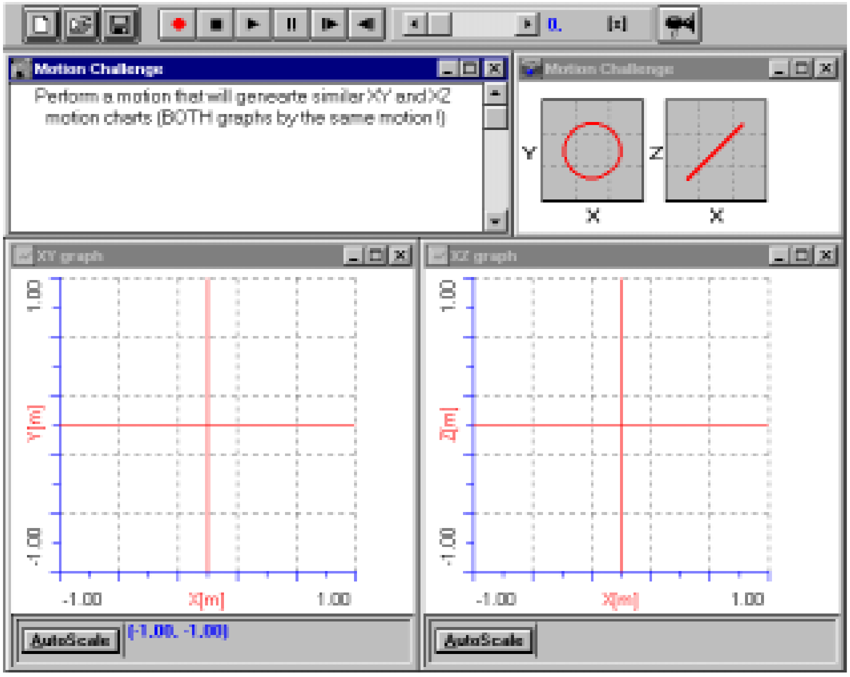

Fig. 4

Two Coupled Pendula

The motion of *two coupled pendula* (*Fig. 5*) is studied.

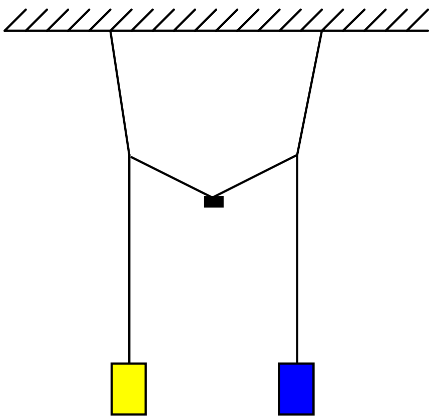

Fig. 5

The yellow body is made to oscillate in the plane of the figure. Then, the (initially stationary) blue body begins to oscillate with increasing amplitude, while the amplitude of the yellow body decreases (*Fig. 6*). At the instant when the motion of the yellow body stops, the energy transfer changes direction.

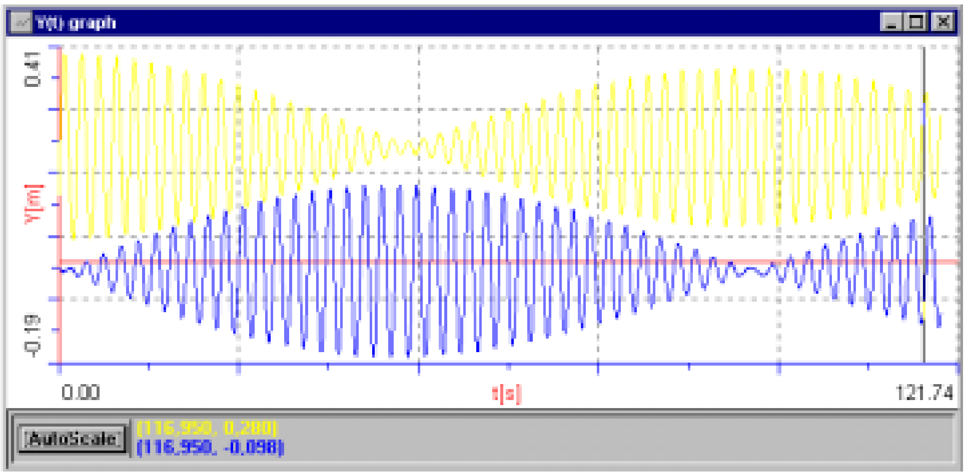

Fig. 6

Magnifying the respective region (*Fig. 7*), the time dependent phase shift between the two motions can clearly be observed. When energy is transferred from the yellow body to the blue one, the oscillation of the latter is late to that of the other by $\pi/2$. As soon as the direction of the energy transfer changes, the sign of the phase shift changes accordingly.

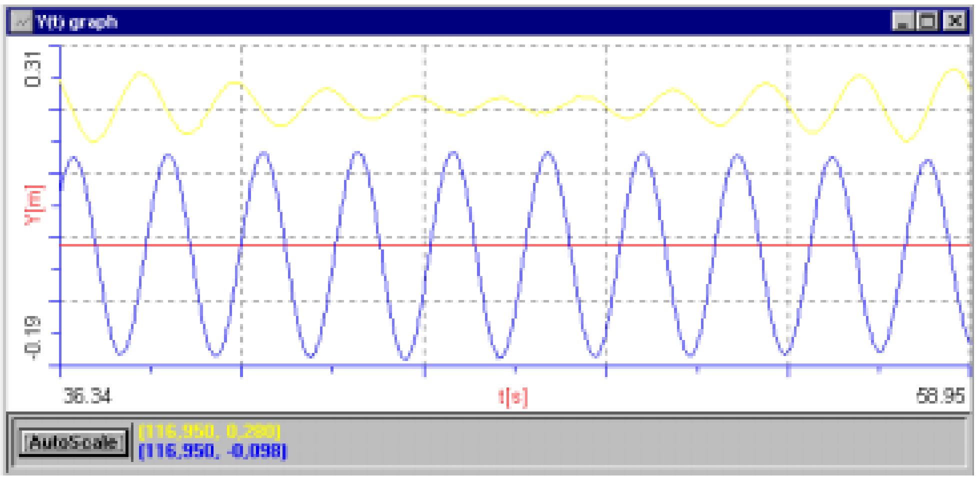

Fig. 7

Another interesting exercise is, when the yellow body is started so as to move on a circular trajectory. In this case, the x and y components of the motion are coupled, however, with different time constants. Traces of this fairly spectacular motion are shown in *Fig. 8*.

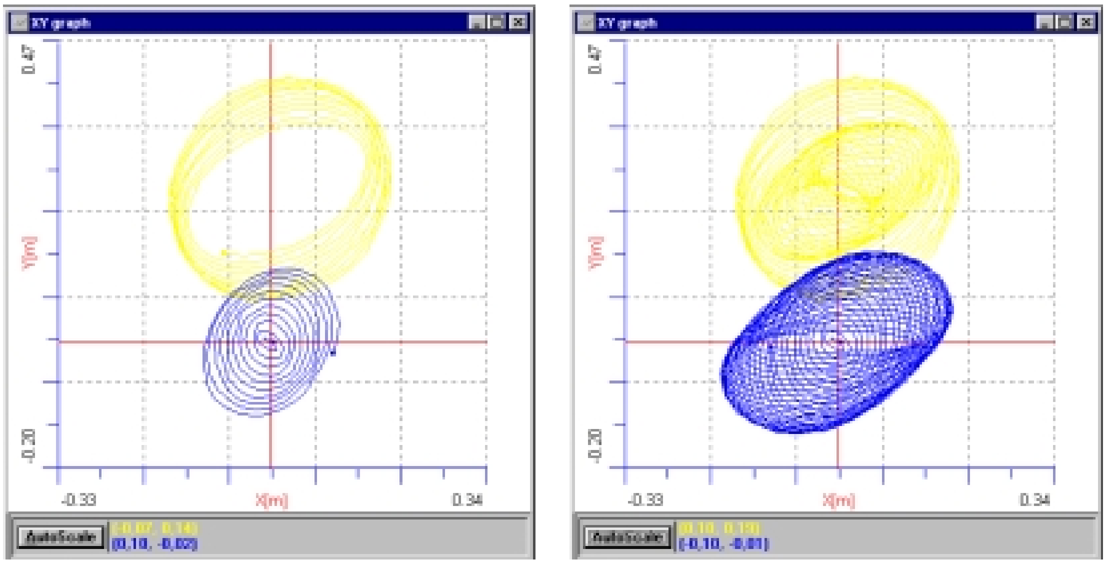

Fig. 8

Torsion Table

Interesting observations can also be made on other simple systems. Before investigating the behaviour of our *torsion table* (used for some time in the laboratory) by the *V-scope*, it was thought that our device obeys an exponential damping. Plotting the angular position of the torsion table as a function of time, however, proves that it's damping is rather linear (*Fig. 9*).

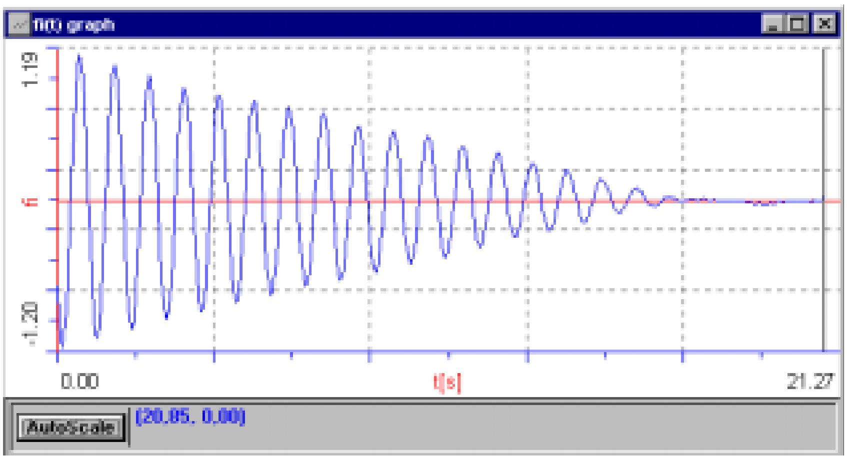

Fig. 9

Chaotic Double Pendulum

In this section we investigate the behaviour of the *chaotic double pendulum*. In a series of measurements we start the system from virtually the same position. We use the V-scope to collect the X and Y co-ordinates of the common axis (yellow button) and of the end of the smaller arm (blue button) relative to the "centre" of the double pendulum (*Fig. 1*). The software *Vscope for Windows* is used to follow essential details of the motion: in the XY graph we replay the motion of both buttons either continuously or step-by-step. We may plot, for example, the relative position of the blue button to the yellow one, or can create a table of time and position data as shown in *Fig. 3*.

The *V-scope* files for the measurements shown during the workshop are available at the site [3]. For further study of the measurements, *Vscope for Windows* can be downloaded from [4] free.

Four *V-scope* graphs, corresponding virtually to the same starting position, are shown in figure (*Fig. 10*).

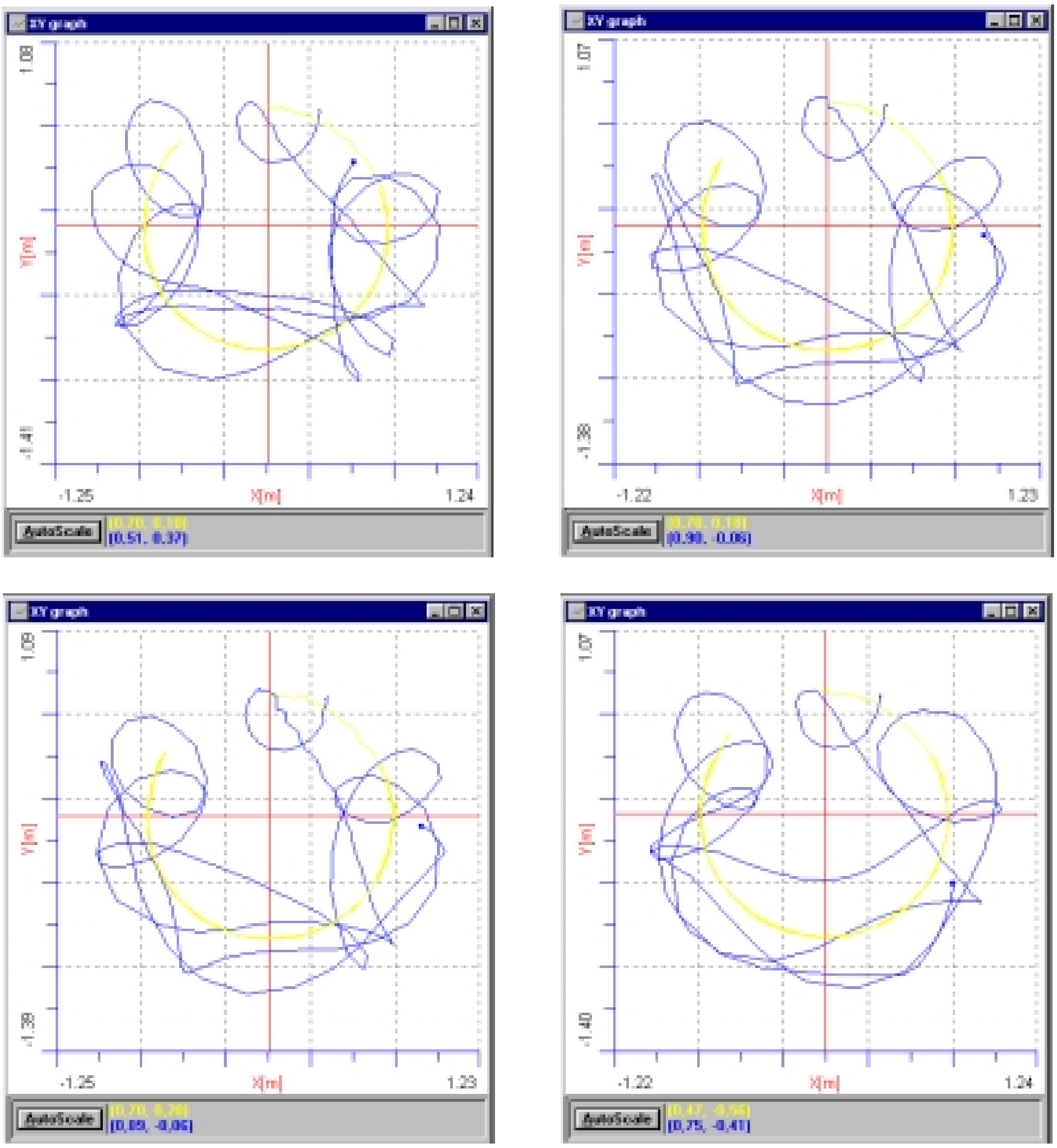

Fig. 10

Note that a comparative study of these records cannot be performed in the framework of *Vscope for Windows*. (The software is able to open one measurement at a time.) A further limitation is, that only simple user defined functions can be created, insufficient for the analysis needed here. Therefore (utilising the respective option of the *Vscope for Windows*), we exported the co-ordinates of buttons and the corresponding times into ASCII files, and developed computer codes for a detailed analysis in the framework of *Matlab 5.1* [5].

Analysis of Measured Data of Chaotic Double Pendulum by Matlab

The motion has been reconstructed from the data t, X_y, Y_y, X_b, Y_b. The respective trajectories are identical to those obtained by V-scope (*Fig. 10*). For inspection, the vertical (Y) co-ordinates of the buttons are plotted as a function of time in *Fig. 11*. The dotted line stands for the end of the larger arm, while the heavy solid line corresponds to the position of the end of the smaller arm relative to the common axis.

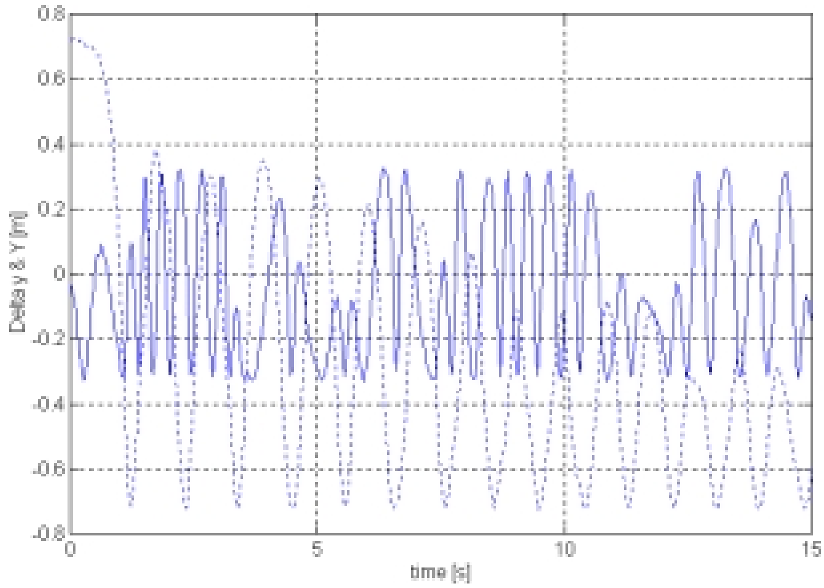

Fig. 11

The oscillatory behaviour of these functions is born out by the respective fast-Fourier-transforms (*Fig. 12*). The dominant frequencies are ~ 0.5 Hz for the larger arm and ~ 2 Hz for the smaller one, that roughly correspond to the eigen frequencies of these arms. These graphs look qualitatively similar for all the four measurements, and give little information of the irregular, unpredictable features of the system.

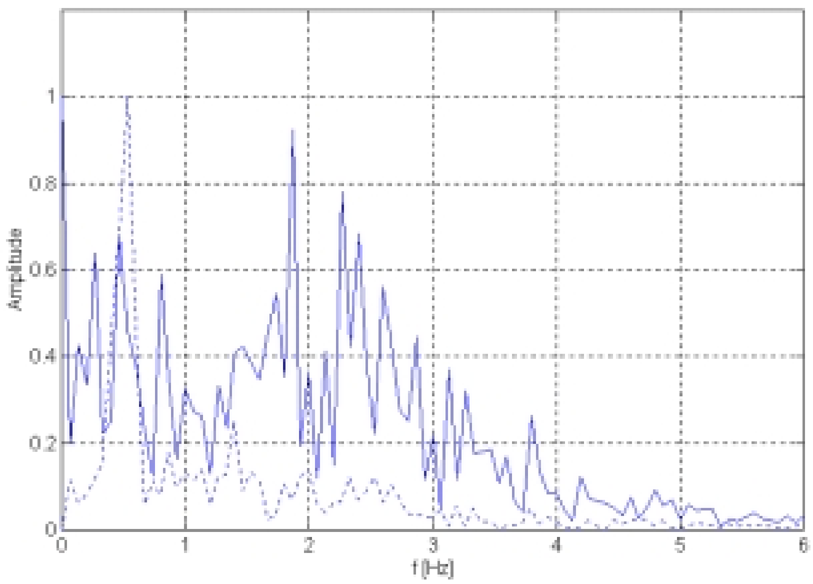

Fig. 12

The chaotic behaviour becomes immediately evident, however, when the *angular velocity* or the *rotation angle* is plotted as a function of time. The former can be calculated from the *X* and *Y* co-ordinates of the buttons, while the latter can be obtained by integrating the angular velocity with respect to time.

The angular velocities of the larger and smaller arms are shown as a function of time in *Figs. 13* and *14*, respectively. (The colours denote different measurements with virtually the same initial conditions.) While the behaviour of the larger arm is fairly regular with some superimposed chaotic component, the motion of the small arm is far less predictable. Although the latter behaves fairly uniformly in the first 2-5 seconds, no similarity can be observed between the motions at later stages.

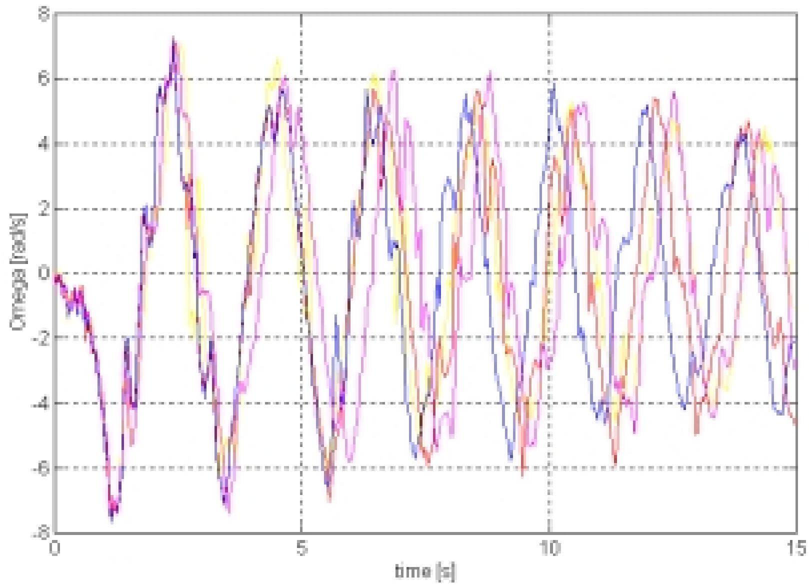

Fig. 13

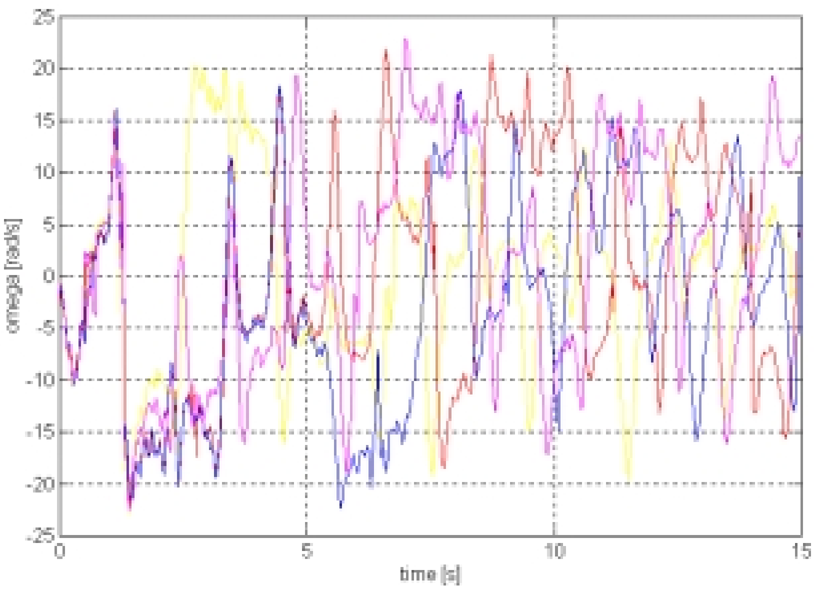

Fig. 14

This is more obvious from the rotation angle vs. time plots (*Figs. 15* and *16*).

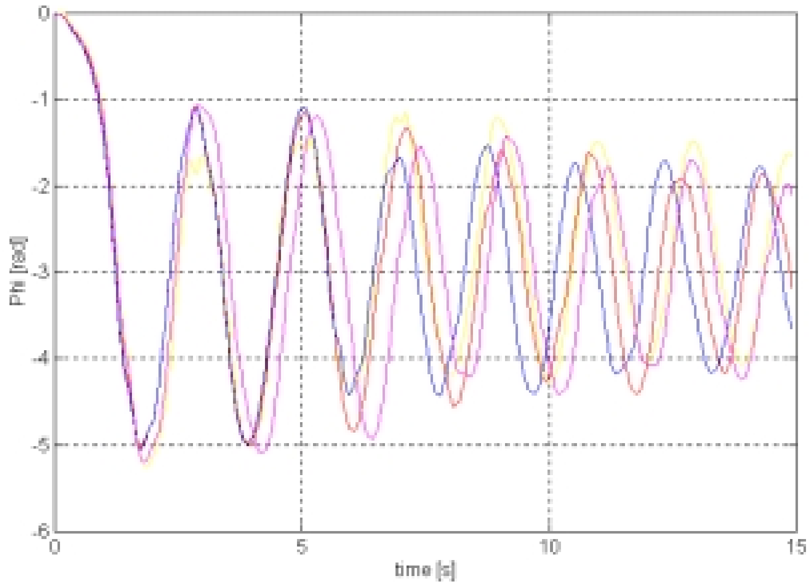

Fig. 15

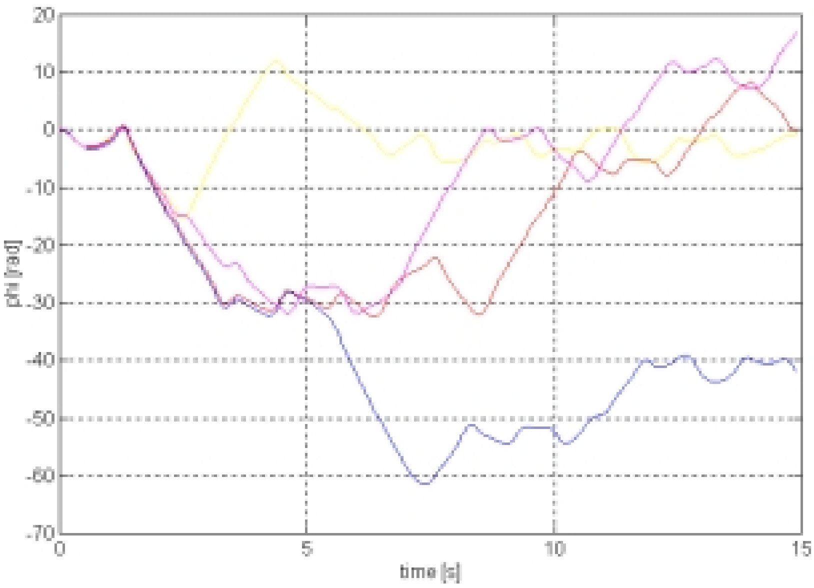

Fig. 16

It has been found that the chaotic behaviour is best displayed by the rotation angle vs. time relationship of the smaller arm. In the first seconds only slight differences that are hardly perceptible appear between the angular positions. A few seconds later, however, the deviation may be as large as 60 rad (~ ten turns!). The transition regime is shown in *Fig. 17* at a higher magnification. The differences develop quite suddenly, at an angular position where the rotation of the smaller arm changes direction.

This behaviour can be better understood if one correlates this graph with the actual spatial position of the arms (as demonstrated during the workshop by a computer animation based on the experimental data). At the instances where the irregular behaviour starts (usually the endpoint of a swing of the larger arm), the smaller arm "hesitates" whether to bounce over the end of the larger arm. Understandably, such critical dynamic situations are able to amplify even small differences, offering a way for chaos to enter.

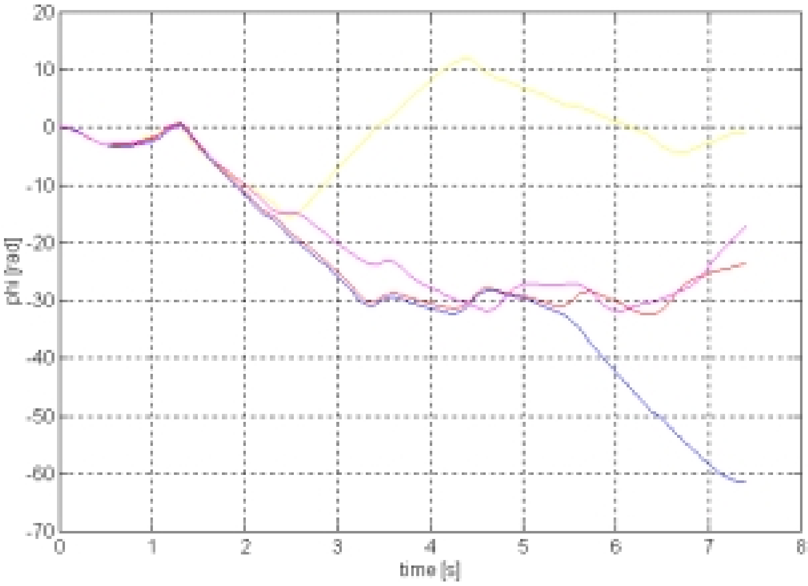

Fig. 17

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