MATHEMATICS-1　　　　**Exam (A)**　　　　**NAME:** ....

18 December 2009

1. (7p)	2. (7p)	3. (12p)	4. (8p)	5. (6p)	6. (6p)	7. (9p)	Th. (5p)	Σ (Max 60p)	MW	Σ+MW

1. Find the following limits: a.) $\lim\limits_{n\to\infty}\left(\dfrac{n+3}{n-1}\right)^{2n}$　b.) $\lim\limits_{n\to\infty}\dfrac{1{,}2^n}{n^{100}}$ (Hint for b: Use quotient rule!)

2. Find the derivative of the following functions

　　a.) by definition: $f(x)=\dfrac{1}{x+3}$　　　　b.) by rules: $g(x)=\dfrac{3x}{\sin(5x)}$

3. Sketch the graph of the function $f(x)=x+1+\dfrac{4}{x-1}$

4. Find the following integrals: a.) $\int\left(2x^4+1\right)\cdot \ln x\ dx$; b.) $\int\dfrac{x+3}{\left(x^2+1\right)\cdot(x+2)}\ dx$

5. Find the volume of the solid given by the rotation of $f(x)=\dfrac{e^x}{e^x+1}$ over $[0;1]$ about the x-axis!

6. Evaluate the following improper integral: $\displaystyle\int_0^\infty\dfrac{2}{x^2+6x+8}\ dx$

7. For which values of a and b has the following system of equations

$$\begin{aligned}x\ +\ 2y\ +\ \ \ z\ &=\ 2\\ 2x\ +\ 5y\ +\ 3z\ &=\ 1\\ -x\ +\ \ y\ +\ a\cdot z\ &=\ b\end{aligned}$$

　　a.)　no solution
　　b.)　exactly one solution
　　c.)　infinitely many solutions

Theoretical question:

Show that the substitution $t=\tan\left(\dfrac{x}{2}\right)$ rationalizes the integral $\displaystyle\int\dfrac{1}{2\sin x-\cos x}dx$