Ex. 2010. Dec. 17.

1.) a.) $\frac{2n^2-1}{n^2+2} \to 2$; $|a_n - A| = \left|\frac{2n^2-1}{n^2+2} - 2\right| = \left|\frac{\cancel{2n^2}-1-\cancel{2n^2}-4}{n^2+2}\right| = \frac{5}{n^2+2} < \frac{3}{100}$

$\Downarrow$

$\boxed{N(\varepsilon)=12} \Leftarrow \overbrace{\sqrt{164{,}7}}^{\sim 12{,}8} < n \Leftarrow 164{,}7 < n^2 \Leftarrow 494 < 3n^2 \Leftarrow 500 < 3n^2+6$

b.) $\left(\frac{3n+1}{3n-1}\right)^{4n} = \left(\frac{3n-1+2}{3n-1}\right)^{4n} = \left[\underbrace{\left(1+\frac{2}{3n-1}\right)^{\frac{3n-1}{2}}}_{\to e}\right]^{\overbrace{\frac{2}{3n-1}\cdot 4n}^{\to \frac{8}{3}}} \to e^{8/3}$

2.) $f(x) = x^3+2x+\sqrt[3]{x+7}$; $x_0 = 1$

$f(x_0) = 1+2+\sqrt[3]{8} = 5$

$f'(x) = 3x^2+2+\frac{1}{3}\cdot(x+7)^{-2/3}\Big|_1 = 3+2+\frac{1}{3}\cdot\frac{1}{(\sqrt[3]{8})^2} = 5+\frac{1}{12} = \frac{61}{12}$

tan. line: $y = 5+\frac{61}{12}(x-1)$

3.) a.) $(\sqrt{x+3})' = \lim_{h\to 0} \frac{\sqrt{x+h+3}-\sqrt{x+3}}{h} \Rightarrow \frac{(\sqrt{x+h+3}-\sqrt{x+3})\cdot(\sqrt{x+h+3}+\sqrt{x+3})}{h\cdot(\sqrt{x+h+3}+\sqrt{x+3})} = \frac{\cancel{x}+h+\cancel{3}-\cancel{x}-\cancel{3}}{h(\sqrt{x+h+3}+\sqrt{x+3})}$

$\sqrt{x+3} = \lim_{h\to 0} \frac{1}{\sqrt{x+h+3}+\sqrt{x+3}} = \frac{1}{2\sqrt{x+3}}$

b.) $\left(\frac{x^3+7x+2}{\cos 2x}\right)' = \frac{(3x^2+7)\cdot\cos 2x+2\cdot\sin 2x(x^3+7x+2)}{\cos^2 2x}$

4.) $f(x) = \frac{x^2-3}{x-2}$; $D_f = \mathbb{R}\setminus\{2\}$; $\lim_{x\to-\infty} f(x) = -\infty$; $\lim_{x\to\infty} f(x) = +\infty$

$f'(x) = \frac{2x(x-2)-(x^2-3)}{(x-2)^2} = \frac{x^2-4x+3}{(x-2)^2}$ $\lim_{x\to 2^-} f(x) = \frac{4-3}{0^-} = -\infty$; $\lim_{x\to 2^+} f(x) = \frac{4-3}{0^+} = +\infty$

$f'(x) = 0$ if $x^2-4x+3 = 0 \Rightarrow x_1 = 1,\ x_2 = 3$

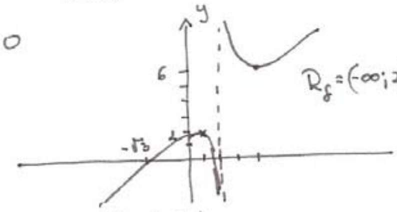
$(x-1)(x-3)$

x	$(-\infty;1)$	1	$(1;2)$	2	$(2;3)$	3	$(3;\infty)$
$f'(x)$	+	0	−	X	−	0	+
$f(x)$	↗	2 max	↘		↘	6 min	↗

$f''(x) = \frac{(2x-4)(x-2)^2-2(x-2)(x^2-4x+3)}{(x-2)^4}$

$f''(x) = 2\cdot\frac{x^2-4x+4-(x^2-4x+3)}{(x-2)^3} = \frac{2}{(x-2)^3} \neq 0$

x	$(-\infty;1)$	1	$(1;\infty)$
$f''(x)$	−	X	+
$f(x)$	∩	X	∪



$R_f = (-\infty;2] \cup [6;\infty)$

5.) $P_1(1;0;0)$; $P_2(1;2;3)$; $P_3(2;1;0)$

$\overrightarrow{P_1P_2} = (0;2;3)$; $\overrightarrow{P_1P_3} = (1;1;0)$

$\underline{n} = \overrightarrow{P_1P_2}\times\overrightarrow{P_1P_3} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 2 & 3 \\ 1 & 1 & 0 \end{vmatrix} = -3\underline{i}+3\underline{j}-2\underline{k}$

plane: $-3(x-1)+3y-2z = 0$

6) a.) $\int(2x+3)\cos x\,dx = (2x+3)\sin x - \int 2\sin x\,dx = (2x+3)\sin x + 2\cos x + C$ 4p

$f = 2x+3$; $g' = \cos x$

$f' = 2$; $g = \sin x$

b.) $\int_{x=0}^{x=\pi/2} \frac{2x-2\sin x}{(x^2+2\cos x)^3}\,dx = \int_{u=2}^{u=\frac{\pi^2}{4}} \frac{1}{u^3}\,du = \left[\frac{u^{-2}}{-2}\right]_2^{\frac{\pi^2}{4}} = -\frac{1}{2}\left(\frac{16}{\pi^4}-\frac{1}{4}\right) = \frac{1}{8}-\frac{8}{\pi^4}$ 5p

$u = x^2+2\cos x \Rightarrow du = (2x-2\sin x)\,dx$

c.) $\int_0^\infty \frac{16}{(x+1)(x+9)}\,dx = \int_0^\infty \frac{2}{x+1}-\frac{2}{x+9}\,dx = \lim_{b\to\infty} 2\cdot\left[\ln\frac{x+1}{x+9}\right]_0^b = \lim_{b\to\infty} 2\cdot\ln\overbrace{\frac{b+1}{b+9}}^{\to 1} - 2\cdot\ln\frac{1}{9}$

$= 2\underbrace{\ln 1}_{0} + 2\ln 9 = \underline{\ln 81}$ 6p

$\frac{1}{(x+1)(x+9)} = \frac{A}{x+1}+\frac{B}{x+9}$

$1 = A(x+9)+B(x+1)$

$x = -1 \Rightarrow 16 = 8A \Rightarrow A = \frac{16}{8}$; $A = 2$; $x = -9 \Rightarrow 16 = -8B$; $B = -2$

7) $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 5 & 5 & 6 \\ -1 & -1 & a & b \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 3 & 0 \\ 0 & 1 & (a+1) & (b+3) \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & (a-2) & (b+3) \end{array}\right]$ 3p

a.) unique sol: if $a \neq 2$

b.) no sol: if $a = 2$; $b \neq -3$ (1+1+1p)

c.) ∞ sol: $a = 2$ and $b = -3$

d.) $a = 1$ and $b = -1 \Rightarrow -z = 2 \Rightarrow z = -2$ (2p)

$y+3z = y-6 = 0 \Rightarrow y = 6$

$x+2y+z = x+12-2 = 3 \Rightarrow x = -7$