

MATHEMATICS-1**Exam (B) NAME:****12. January 2011**

1. (6p)	2. (4p)	3. (7p)	4.(10p)	5. (6p)	6. (15p)	7. (8p)	Th. (4p)	Σ (Max 60p)

Corrected by:

1. a.) Find $N(\varepsilon)$ for the sequence $a_n = \frac{3n^2 - 1}{5n^2 + 1}$ if $\varepsilon = 0,02$.

b.) Find the following limit: $\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 3n - 1} - \sqrt{n^2 + n + 2} \right)$

2. Find the equation of the tangent line to $f(x) = x^2 + 3x + \sqrt[4]{x+17}$ at the point $x_0 = -1$!

3. Find the derivative of the following functions

a.) by definition: $f(x) = x^2 + 3x - 2$

b.) by rules: $g(x) = \frac{e^{2x} + \ln(x+3)}{\sin(2x+1)}$

4. Sketch the graph of the function $f(x) = \frac{(x+3)^2}{x+1}$

5. Find the volume of the solid given by the rotation of $f(x) = \sqrt{x} \cdot e^x$ over $[0;1]$ about the x -axis!

6. Find the following integrals:

a.) $\int (2x+3) \cdot \ln x \, dx$; b.) $\int_0^1 \frac{3\sqrt{x} - 3e^{3x}}{(2\sqrt{x^3} - e^{3x})^2} \, dx$; c.) $\int_0^\infty \frac{3}{(x+1)(x+4)} \, dx$

7. For which values of a and b has the following system of equations

$$x + 4y + 2z = 3$$

$$-x - 5y + z = 2$$

$$-2x - 11y + a \cdot z = b$$

d.) exactly one solution

e.) no solution

f.) infinitely many solutions

Find the solution of the system if $a = 2$ and $b = -1$!

Theoretical question:

Show that the substitution $t = \tan\left(\frac{x}{2}\right)$ rationalizes the integral $\int \frac{1}{3 \sin x} dx$