MATHEMATICS-1 **Exam (A)** **NAME:**

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12 January 2011

1. (6p)	2. (4p)	3. (7p)	4.(12p)	5. (4p)	6. (15p)	7. (8p)	Th. (4p)	Σ (Max 60p)

Corrected by: ……

1. Find the following limits:

a.) $\lim\limits_{n\to\infty}\left(\dfrac{5n+2}{5n-1}\right)^{2n}$ b.) $\lim\limits_{n\to\infty}\left(\sqrt{n^2+4n+3}-n\right)$

2. Find the equation of the tangent line to $f(x)=2x^5-3x+\sqrt{x+5}$ at the point $x_0=-1$!

3. Find the derivative of the following functions

a.) by definition: $f(x)=\dfrac{2}{3x+1}$ b.) by rules: $g(x)=\dfrac{x^4+e^{2x}}{3x-2}$

4. Sketch the graph of the function $f(x)=\dfrac{x+1}{(x+3)^2}$

5. Find the equation of the line passing through the points $P_1=(1;2;0)$ and $P_2=(3;1;5)$

6. Find the following integrals:

a.) $\int(2x+5)\cdot e^{2x}\,dx$; b.) $\int\limits_1^e \dfrac{3x^2+x^{-1}}{\left(x^3+\ln x\right)^4}\,dx$; c.) $\int\limits_1^\infty \dfrac{5}{x^2+5x}\,dx$

7. For which values of a and b has the following system of equations

$$\begin{aligned} x &+ 4y + 2z = 8 \\ 3x &+ 14y + 4z = 18 \\ -x &- 2y + a\cdot z = b \end{aligned}$$

a.) exactly one solution
b.) no solution
c.) infinitely many solutions

Find the solution of the system if $a=-1$ and $b=1$!

Theoretical question:

Show that the substitution $t=\tan\left(\dfrac{x}{2}\right)$ rationalizes the integral $\int\dfrac{1}{\cos x}dx$