Theoretical questions

(1.) $P_0 = (x_0, y_0, z_0)$ is a given point in the plane, and $\underline{n} = n_x \underline{i} + n_y \underline{j} + n_z \underline{k}$ vector is perpendicular to the plane.

Let be $P = (x, y, z)$ any arbitrary point in the plane.

Then $\overrightarrow{P_0P} \perp \underline{n}$ so $\overrightarrow{P_0P} \cdot \underline{n} = 0 \Rightarrow n_x(x-x_0) + n_y(y-y_0) + n_z(z-z_0) = 0$ is the equation of the plane.

(2.) The line is parallell with the vector $\underline{v} = v_x \underline{i} + v_y \underline{j} + v_z \underline{k}$, and $P_0 = (x_0, y_0, z_0)$ is on the line.

Let be $P(x, y, z)$ any arbitrary point on the line.

Then $\overrightarrow{P_0P} \parallel \underline{v}$, so $\overrightarrow{P_0P} = t \cdot \underline{v}$ ($\overrightarrow{P_0P}$ is a multiple of $\underline{v}$)

that means $(x-x_0)\underline{i} + (y-y_0)\underline{j} + (z-z_0)\underline{k} = t \cdot (v_x \underline{i} + v_y \underline{j} + v_z \underline{k})$

(3.) Let be $f(x)$ and $g(x)$ continuously differentiable functions.

That means, f, g, f', g' and their products are integrable.

We know from the chain rule: $[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + g'(x) \cdot f(x)$.

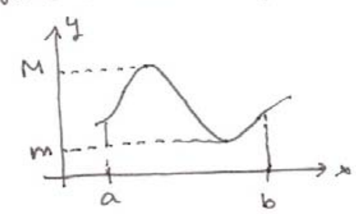
Integrating both sides we get: $\underbrace{\int [f(x) \cdot g(x)]' dx}_{f(x) \cdot g(x)} = \underbrace{\int f'(x) g(x) + g'(x) f(x)\, dx}_{= \int f'(x) g(x)\, dx + \int f(x) g'(x)\, dx}$

$$\boxed{\int f(x) \cdot g'(x) = f(x) \cdot g(x) - \int f'(x) g(x)\, dx} \Longleftarrow$$

(4.) $f(x)$ is Riemann integrable on $[a;b]$ means: $\lim\limits_{\substack{n \to \infty \\ (\Delta x_k)_{max} \to 0}} \sum\limits_{k=1}^{n} f(\xi_k) \Delta x_k$ exists, and the limit doesn't depend on the „place" of ξ_k in $[x_{k-1}; x_k]$

$$\int_a^b c \cdot f(x)\, dx = \lim_{\substack{n \to \infty \\ (\Delta x_k)_{max} \to 0}} \sum_{k=1}^{n} c \cdot f(\xi_k) \Delta x_k = c \cdot \lim_{\substack{n \to \infty \\ (\Delta x_k)_{max} \to 0}} \sum_{k=1}^{n} f(\xi_k) \Delta x_k = c \cdot \int_a^b f(x)\, dx$$

(5.) Let be m the smallest, M the biggest function value of the continuous $f(x)$ on $[a;b]$



then
$$m \cdot (b-a) \leq \int_a^b f(x)\, dx \leq M(b-a)$$
$$m \leq \frac{1}{b-a} \cdot \int_a^b f(x)\, dx \leq M$$

Because $\frac{1}{b-a} \cdot \int_a^b f(x)\, dx$ is a number between m and M, it is a function value, as well $\Rightarrow \boxed{\frac{1}{b-a} \int_a^b f(x)\, dx = f(\xi)}$; $\xi \in [a;b]$