Pract. ex. 2,

1. a) $P_1(1;3;5)$; $\underline{n} = \underline{a} = 3\underline{i} + 5\underline{j} + 0\underline{k}$ $\Rightarrow$ plane: $\boxed{3(x-1)+5(y-3)=0}$

b) $\overrightarrow{P_1P_2} = 0\underline{i} + 2\underline{j} + 3\underline{k}$, $\overrightarrow{P_1P_3} = 1\underline{i} - 2\underline{j} + 1\underline{k}$ $\Big\}$ $\underline{n} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 2 & 3 \\ 1 & -2 & 1 \end{vmatrix} = 8\underline{i} + 3\underline{j} - 2\underline{k}$ $\Rightarrow$ plane: (using P_1) $\boxed{8(x-1)+3(y-3)-2(z-5)=0}$

c) $P_1(1;3;5)$; $\underline{v} = \underline{a} = 3\underline{i} + 5\underline{j}$ $\Rightarrow$ line: $\boxed{x = 1+3t;\ y = 3+5t;\ z = 5}\ (+0t)$

d) $\underline{v} = \overrightarrow{P_1P_2} = 0\underline{i} + 2\underline{j} + 3\underline{k}$ $\Rightarrow$ line: $\boxed{x = 1;\ y = 3+2t;\ z = 5+3t}$

2. a) $\int 3x^2 + \sqrt[4]{2+x} + \cos x \, dx = x^3 + \frac{(2+x)^{5/4}}{5/4} + \sin x + C$

b) $\int \sin(x^5+e^x) \cdot (5x^4+e^x)\,dx = \int \sin\ dt = -\cos t + C = -\cos(x^5+e^x) + C$

$t = x^5 + e^x \Rightarrow \frac{dt}{dx} = 5x^4 + e^x \Rightarrow dt = (5x^4+e^x)dx$

3. a) $\int (x^2+3)\cdot e^x\,dx = (x^2+3)\cdot e^x - \int 2x\, e^x dx = (x^2+3)\cdot e^x - 2x\cdot e^x + \int 2e^x dx = e^x[x^2+3-2x+2] + C$

$f = x^2+3$; $g' = e^x$ $\quad$ $f = 2x$; $g' = e^x$

$f' = 2x$; $g = e^x$ $\quad$ $f' = 2$; $g = e^x$

b) $\int (x+2)\cdot \ln x = \left(\frac{x^2}{2} + 2x\right)\cdot \ln x - \int \underbrace{\left(\frac{x^2}{2}+2x\right)\cdot\frac{1}{x}}_{\frac{x}{2}+2} dx = \left(\frac{x^2}{2}+2x\right)\cdot \ln x - \frac{x^2}{4} - 2x + C$

$f = \ln x$; $g' = x+2$

$f' = \frac{1}{x}$; $g = \frac{x^2}{2} + 2x$

c) $\int e^x \cdot \cos 2x \, dx = e^x\cdot\cos 2x + \int 2\sin 2x \cdot e^x\,dx = e^x\cdot\cos 2x + 2e^x\cdot\sin 2x - 4\int \cos 2x\cdot e^x dx$

$f = \cos 2x$; $g' = e^x$ $\quad$ $f = 2\sin 2x$; $g' = e^x$ $\quad$ $y = e^x\cdot[\cos 2x + 2\sin 2x] - 4y$

$f' = -2\sin 2x$; $g = e^x$ $\quad$ $f' = 4\cos 2x$; $g = e^x$ $\quad$ $\Downarrow$

$y = \frac{e^x}{5}\cdot[\cos 2x + 2\sin 2x] + C$

4. a) $\int \frac{2}{x^2+2x}dx = \int \frac{A}{x} + \frac{B}{x+2}\,dx = \int \frac{1}{x} - \frac{1}{x+2}\,dx = \ln|x| - \ln|x+2| + C$

$2 = A(x+2) + Bx$

$x = 0 \Rightarrow 2 = 2A \Rightarrow A = 1$

$x = -2 \Rightarrow 2 = -2B \Rightarrow B = -1$

b) $\int \frac{x^2+x+2}{(x+1)^2(x+3)}\,dx = \int \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+3}dx = \int \frac{-1}{x+1} + \frac{1}{(x+1)^2} + \frac{2}{x+3}\,dx =$

$= -\ln|x+1| - \frac{1}{x+1} + 2\ln|x+3| + \text{const}$

$x^2+x+2 = A(x+1)(x+3) + B(x+3) + C(x+1)^2$

$x = -1 \Rightarrow 2 = 2B \Rightarrow B = 1$

$x = -3 \Rightarrow 8 = 4C \Rightarrow C = 2$

$x = 0 \Rightarrow 2 = 3A + 3B + C \Rightarrow 2 = 3A + 5 \Rightarrow A = -1$

c) $\int \frac{3x^2 - 7x + 7}{(x-2)(x^2+1)}dx = \int \frac{A}{x-2} + \frac{Bx+C}{x^2+1}dx = \int \frac{1}{x-2} + \frac{2x-3}{x^2+1}\,dx = \int \frac{1}{x-2} + \frac{2x}{x^2+1} - \frac{3}{x^2+1}\,dx =$

$= \ln|x-2| + \ln(x^2+1) - 3\tan^{-1}x + \text{const}$

$3x^2 - 7x + 7 = A(x^2+1) + (Bx+C)(x-2)$

$x = 2 \Rightarrow 5 = 5A \Rightarrow A = 1$

$x = 0 \Rightarrow 7 = 1 - 2C \Rightarrow C = -3$

$x = 1 \Rightarrow 3 = 2 - B + 3 \Rightarrow B = 2$

(5) a.) $\displaystyle\int \frac{1}{2\sqrt{x}-\sqrt[4]{x}}\,dx = \int \frac{4t^3}{t^2-t}\,dt = \int \frac{4t^2}{t-1}\,dt = 4\int \frac{t^2-1+1}{t-1}\,dt = 4\int t+1+\frac{1}{t-1}\,dt =$

$t = \sqrt[4]{x} \Rightarrow t^2 = \sqrt{x}$

$x = t^4 \Rightarrow dx = 4t^3\,dt$

$\displaystyle = 4\left(\frac{t^2}{2} + t + \ln|t-1|\right) + C = 2\sqrt{x} + 4\sqrt[4]{x} + 4\ln|\sqrt[4]{x}-1| + C$

b.) $\displaystyle\int \frac{e^x}{e^{2x}+6e^x+8}\,dx = \int \frac{1}{t^2+6t+8}\,dt = \int \frac{1}{(t+2)(t+4)}\,dt = \int \frac{A}{t+2} + \frac{B}{t+4}\,dt =$

$t = e^x \Rightarrow dt = e^x dx$

$1 = A(t+4) + B(t+2)$

$t=-4 \Rightarrow \quad 1 = -2B \Rightarrow B = -1/2$

$t=-2 \Rightarrow \quad 1 = 2A \Rightarrow A = 1/2$

$\displaystyle = \frac{1}{2}\int \frac{1}{t+2} - \frac{1}{t+4}\,dt =$

$\displaystyle = \frac{1}{2}\left(\ln|t+2| - \ln|t+4|\right) + C =$

$\displaystyle = \frac{1}{2}\left(\ln|e^x+2| - \ln|e^x+4|\right) + C$

(6.) $f(x) = g(x): \quad \frac{16}{x^2} = x^2 \Rightarrow x^4 = 16 \Rightarrow x = \pm 2$; only $x = 2 \in [1;4]$.

$\displaystyle \text{Area} = \int_1^4 \left|\frac{16}{x^2} - x^2\right| dx = \int_1^2 \frac{16}{x^2} - x^2\,dx + \int_2^4 -\frac{16}{x^2} + x^2\,dx = \left[-\frac{16}{x} - \frac{x^3}{3}\right]_1^2 + \left[\frac{16}{x} + \frac{x^3}{3}\right]_2^4 =$

$\displaystyle = \left(-\cancel{8} - \frac{8}{3} + \cancel{16} + \frac{1}{3}\right) + \left(16 + \frac{64}{3} - \cancel{8} - \frac{8}{3}\right) = 16 + \frac{48}{3} + \frac{1}{3} = 32\tfrac{1}{3}$ sq. un.

(7.) $\displaystyle V = \pi\int_1^3 \left(\frac{1}{x}\right)^2 dx = \pi\cdot\left[-\frac{1}{x}\right]_1^3 = \pi\cdot\left(-\frac{1}{3}+1\right) = \frac{2\pi}{3}$ vol. un.

(8.) $f(x) = \ln(\sin x) \qquad \displaystyle s = \int_{\pi/3}^{\pi/2} \sqrt{\frac{1}{\sin^2 x}}\,dx = \int_{x=\pi/3}^{\pi/2} \frac{1}{\sin x}\,dx = \int_{1/\sqrt{3}}^{1} \frac{1}{t}\,dt = \left[\ln|t|\right]_{1/\sqrt{3}}^{1} = \ln\sqrt{3}$ l. un.

$\displaystyle f'(x) = \frac{1}{\sin x}\cdot\cos x$

$\displaystyle 1 + [f'(x)]^2 = 1 + \frac{\cos^2 x}{\sin^2 x} = \frac{\sin^2 x + \cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$t = \tan\frac{x}{2}$

$\displaystyle dx = \frac{2}{1+t^2}\,dt; \quad \sin x = \frac{2t}{1+t^2}$

(9.) $\displaystyle A_s = 2\pi\cdot\int_0^2 \cosh x\cdot\sqrt{1+\sinh^2 x}\,dx = 2\pi\cdot\int_0^2 \cosh^2 x\,dx = 2\pi\cdot\frac{1}{4}\cdot\int_0^2 (e^x + e^{-x})^2\,dx =$

$(\cosh x)' = \sinh x$

$\displaystyle = \frac{\pi}{2}\cdot\int_0^2 e^{2x} + 2 + e^{-2x}\,dx = \frac{\pi}{2}\cdot\left[\frac{e^{2x}}{2} + 2x + \frac{e^{-2x}}{-2}\right]_0^2 = \frac{\pi}{4}\left[e^4 + 8 - e^{-4}\right]$

(10.) a.) $\displaystyle \int_1^2 \frac{1}{\sqrt{x-1}}\,dx = \lim_{\varepsilon\to 0^+} \int_{1+\varepsilon}^{2} (x-1)^{-1/2}\,dx = \lim_{\varepsilon\to 0^+} \left[2\cdot\sqrt{x-1}\right]_{1+\varepsilon}^{2} = \lim_{\varepsilon\to 0^+} 2(1-\sqrt{\varepsilon}) = 2$

b.) $\displaystyle \int_1^\infty \frac{1}{x^2+3x+2}\,dx = \lim_{b\to\infty} \int_1^b \frac{1}{(x+1)(x+2)}\,dx = \lim_{b\to\infty} \int_1^b \frac{1}{x+1} - \frac{1}{x+2}\,dx =$

$\displaystyle = \lim_{b\to\infty} \left[\ln|x+1| - \ln|x+2|\right]_1^b = \lim_{b\to\infty}\left[\ln(b+1) - \ln(b+2) - (\ln 2 - \ln 3)\right] =$

$\displaystyle = \lim_{b\to\infty} \underbrace{\ln \overset{\to 1}{\frac{b+1}{b+2}}}_{\to \ln 1 = 0} + \ln\frac{3}{2} = \ln\frac{3}{2}$