Practicing exercises-2

1. Let be given the points $P_1=(1, 3, 5)$, $P_2=(1, 5, 8)$, $P_3=(2, 1, 6)$, and the vector $\underline{a} = 3\underline{i} + 5\underline{j}$.
   a.) Find the equation of the plane passing through P_1 and perpendicular to the vector $\underline{a}$!
   b.) Find the equation of the plane passing through the points P_1, P_2 and P_3 !
   c.) Find the equation of the line passing through P_1 and parallell with the vector $\underline{a}$!
   d.) Find the equation of the line passing through P_1 and P_2 !

2. Find the following integrals: $\int 3x^2 + \sqrt[4]{2+x} + \cos x \, dx$; $\int \sin\left(x^5 + e^x\right)\cdot\left(5x^4 + e^x\right) dx$;

3. Find the following integrals: $\int \left(x^2 + 3\right)\cdot e^x \; dx$; $\int (x+2)\cdot \ln x \; dx$; $\int e^x \cdot \cos 2x \; dx$

4. Find the following integrals: $\int \frac{2}{x^2 + 2x} \; dx$; $\int \frac{x^2 + x + 2}{(x+1)^2 \cdot (x+3)} \; dx$; $\int \frac{3x^2 - 7x + 7}{(x-2)\cdot\left(x^2+1\right)} \; dx$

5. Find the following integrals:

   a.) $\int \frac{1}{2\sqrt{x} - \sqrt[4]{x}} \; dx$ (hint: substitute $t = \sqrt[4]{x}$) ; b.) $\int \frac{e^x}{e^{2x} + 6e^x + 8} \; dx$ (hint: substitute $t = e^x$)

6. Find the area between $f(x) = \frac{16}{x^2}$ and $g(x) = x^2$ over the interval [1; 4] !

7. Find the volume of the solid given by the rotation of $f(x) = \frac{1}{x}$ over [1; 3] about the *x*-axis!

8. Find the arc length of the function $f(x) = \ln(\sin x)$ over the interval $\left[\frac{\pi}{3};\frac{\pi}{2}\right]$!

9. Find the surface of the revolution given by rotating about the *x* axis $f(x)=\cosh x$ over [0;2] !

10. Find the following improper integrals: a.) $\int_1^2 \frac{1}{\sqrt{x-1}} \; dx$; b.) $\int_1^\infty \frac{1}{x^2 + 3x + 2} \; dx$

Theoretical questions

1.) Show that the equation of the plane passing through the point $P_0=(x_0,y_0,z_0)$ and perpendicular to the vector $\underline{n} = n_x\underline{i} + n_y\underline{j} + n_z\underline{k}$ is $n_x(x-x_0) + n_y(y-y_0) + n_z(z-z_0) = 0$

2.) Show that the equation of the line passing through the point $P_0=(x_0,y_0,z_0)$ and parallell with the vector $\underline{v} = v_x\underline{i} + v_y\underline{j} + v_z\underline{k}$ is $(x-x_0)\underline{i} + (y-y_0)\underline{j} + (z-z_0)\underline{k} = t\cdot\underline{v}$, where $t \in \Re$

3.) Let be $f(x)$ and $g(x)$ continuously differentiable functions.

Show that $\int f(x)\cdot g'(x)\,dx = f(x)\cdot g(x) - \int f'(x)\cdot g(x)\,dx$

4.) Let be the function $f(x)$ Riemann integrable on the interval $[a;b]$, and c an arbitrary real number.

Show that $\int_a^b c\cdot f(x)\,dx = c\cdot\int_a^b f(x)\,dx$

5.) Let be the function $f(x)$ continuous on the interval $[a;b]$.

Show that there exists $\xi \in [a;b]$ such that $\int_a^b f(x)\,dx = f(\xi)\cdot(b-a))$

Solutions are downloadable from http://knight.kit.bme.hu/Maths
or from http://goliat.eik.bme.hu/~kollarne/Maths/