Practicing exercises

1. Find $N(\varepsilon)$ for the given sequences if $\varepsilon=0{,}01$: $a_n=\frac{1}{n^2+2}$; $b_n=\frac{2n+1}{n+3}$; $c_n=\sqrt{9+\frac{1}{n}}$

2. Find the following limits: $\lim_{n\to\infty}\frac{n^3+\sqrt{n}}{2n-5n^2}$; $\lim_{n\to\infty}\frac{2^n}{n^{20}}$; $\lim_{n\to\infty}\frac{3^n}{n!}$; $\lim_{n\to\infty}\left(\frac{3n+5}{3n-2}\right)^{4n}$; $\lim_{n\to\infty}\sqrt[n]{n+3}$

3. $\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)=?$ $\lim_{x\to-\infty}\left(\sqrt{x^2+x}-x\right)=?$ $\lim_{x\to0}\frac{3x}{\sin x}=?$ $\lim_{x\to0}\frac{\tan x}{\sin 5x}=?$ $\lim_{x\to-3}\frac{x^2-9}{x+3}=?$

4. Find the points of discontinuity for the following functions, and classify them:

$$f(x)=\begin{cases}x+3 & if \quad x<=1\\ \frac{x^2-1}{x-1} & if \quad x>1\end{cases} \quad ; \quad f(x)=\begin{cases}3x^2 & if \quad x<=0\\ \frac{\sin^2 x}{x} & if \quad x>0\end{cases} \quad ; \quad f(x)=\begin{cases}\frac{x}{x-3} & if \quad x<3\\ 2x & if \quad x>=3\end{cases}$$

5. Find the derivative of the following functions

a.) by the definition of the derivative: $f(x)=x^2-2x$; $f(x)=\sqrt{x+3}$; $f(x)=\frac{2}{x}$

b.) by the rules: $f(x)=x^2\cdot\sin(2x)$; $f(x)=\frac{\cos x}{x+\ln x}$; $f(x)=\sqrt{x^3+5x+2^x}$

6. Give the equation of the tangent line to $f(x)=x^4-3x$ at the point x_0=1!

7. Find the following limits by L'Hospital rule:

$$\lim_{x\to0}\frac{1-\cos^2 x}{3x^2} \quad ; \quad \lim_{x\to\infty}\frac{x^2-5x+12}{x-3x^2} \quad ; \quad \lim_{x\to2}\frac{x^2-4}{x^2-5x+6} \quad ; \quad \lim_{x\to0^+}\frac{\ln x}{x^{-2}}$$

8. Find the intervals of monotonity and local extrema for $f(x)=x^2\ln x$

9. Find the intervals of convexity, concavity and inflection points for $f(x)=x^4-10x^3+36x^2+5$

10. Sketch the graph of $f(x)=x^2\ln x$

11. a.) Find x if $\begin{vmatrix}2 & x & 1\\ 0 & 1 & 1\\ 3 & 0 & 2\end{vmatrix}=5$ b.) Evaluate $\underline{a}\cdot\underline{b}$, $cos\phi$, and $\underline{a}\times\underline{b}$ if $\begin{cases}\underline{a}=3\underline{i}+2\underline{j}+\underline{k}\\ \underline{b}=4\underline{i}-\underline{j}+5\underline{k}\end{cases}$

Theoretical questions

1.) Show that $\lim_{x\to0^+}\frac{\sin x}{x}=1$

2.) Prove: If a function is differentiable at x_0, then it is continuous here

3.) Using the definition of the derivative, show that for the differentiable functions *f(x)* and *g(x)* $[f(x)+g(x)]'=f'(x)+g'(x)$

4.) Using $\left[\tan\left(\tan^{-1}x\right)\right]'=x$, and the chain rule, show that $\left(\tan^{-1}x\right)'=\frac{1}{1+x^2}$

5.) Prove that $cosh^2x-sinh^2x=1$!