

An Experimental Problem of a Competition Discussed in a Secondary School Workshop

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Inventing a new experimental problem is not easy.

How can be used the apparatus later?

- to prepare students for other competitions
- as an measuring exercise for inquiring students
 - as a problem of a local competition
 - to discuss the problem in a school workshop**

An experimental problem

in the competition

- limited time
- working alone
- pocket calculator, ruler, graph paper
- limited knowledge (without books)

in the workshop

- more time
- working together
- data analysis by PC software's
- background information

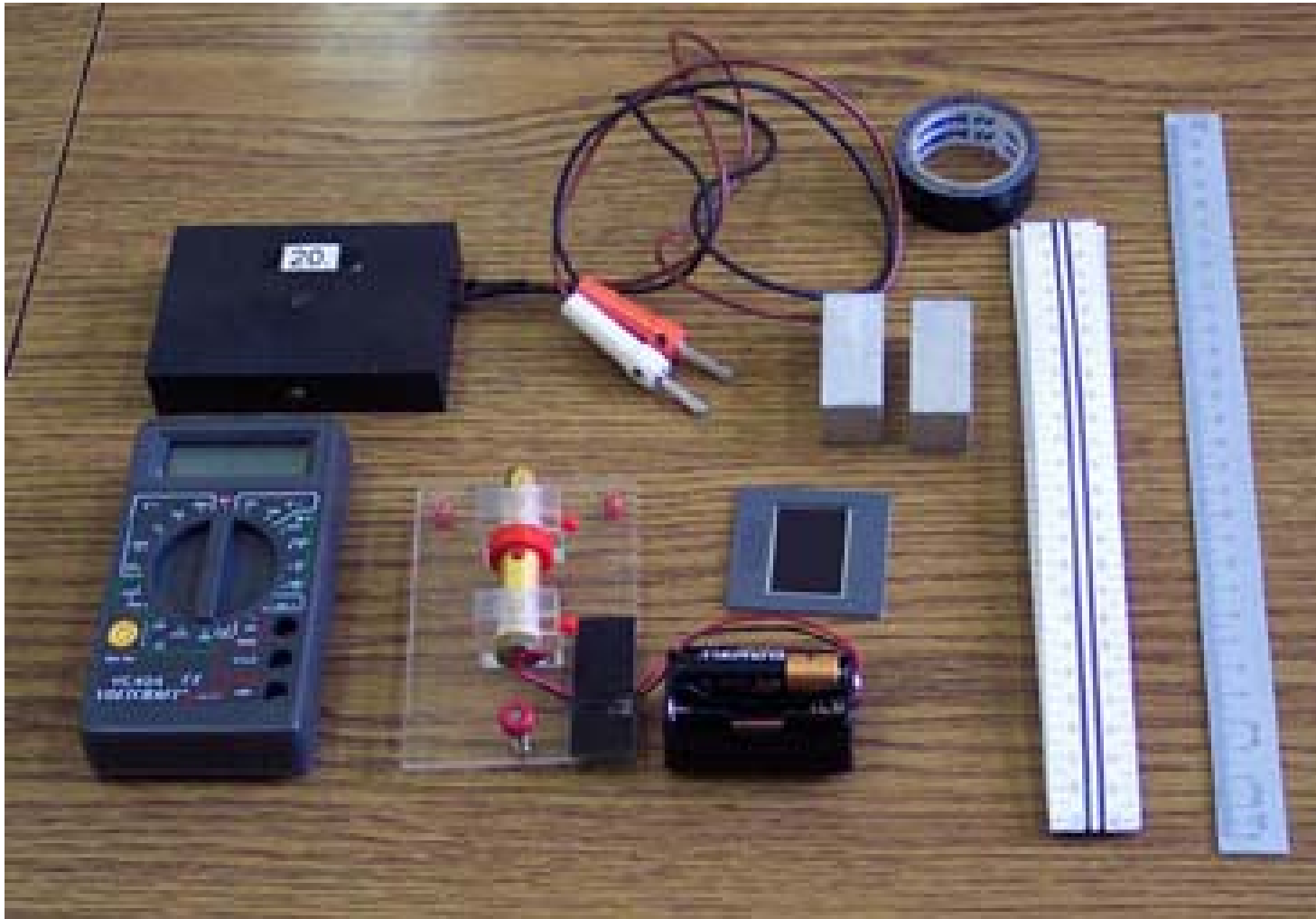
The problem

There are **two optical structures** to investigate by semiconductor laser.

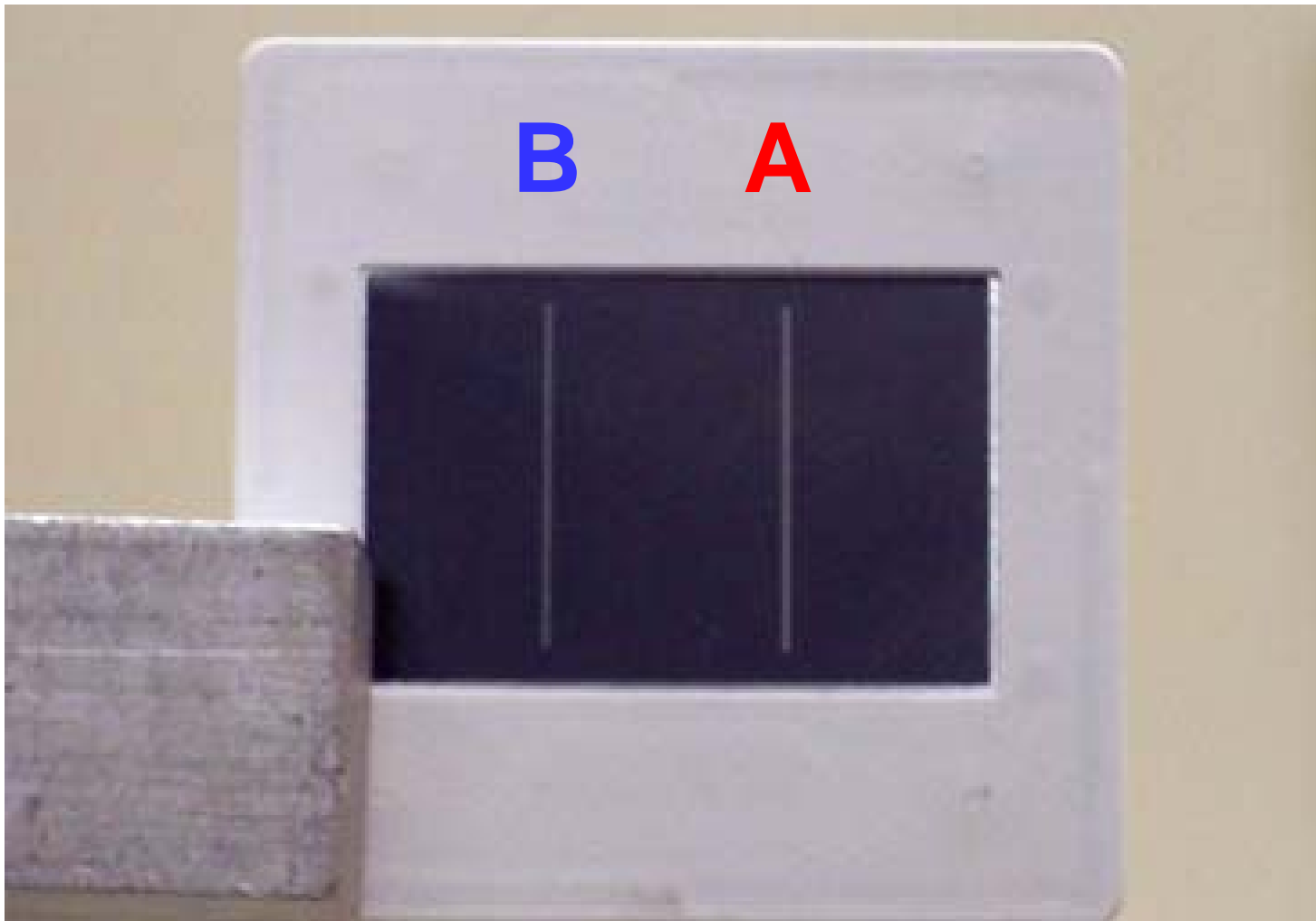
Both of them are **multiple slits**, i.e. a few parallel and identical transparent slits on a dark background separated by the same distance.

From the diffraction pattern determine the **distance**, the **number** and the **width** of the slits in both optical structures.

The apparatus



The slit structures



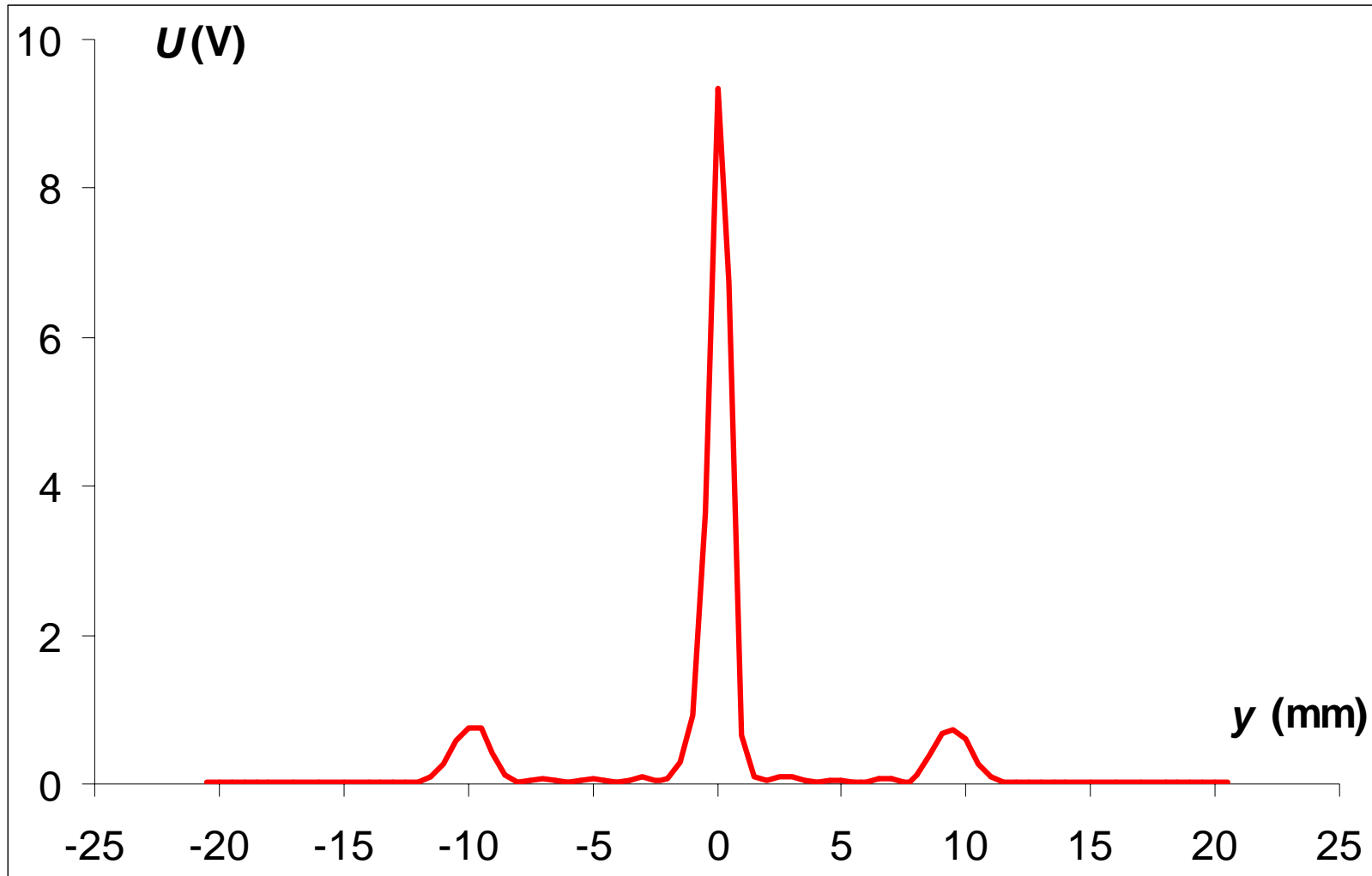
The difficulties of the measurement

- The careful adjusting of the arrangement
- Reading detector position with half a mm accuracy
- Changing the range of the voltmeter during the measurement
- Measuring the background light intensity
- Measuring the light intensity across the „black” background of the slit structures

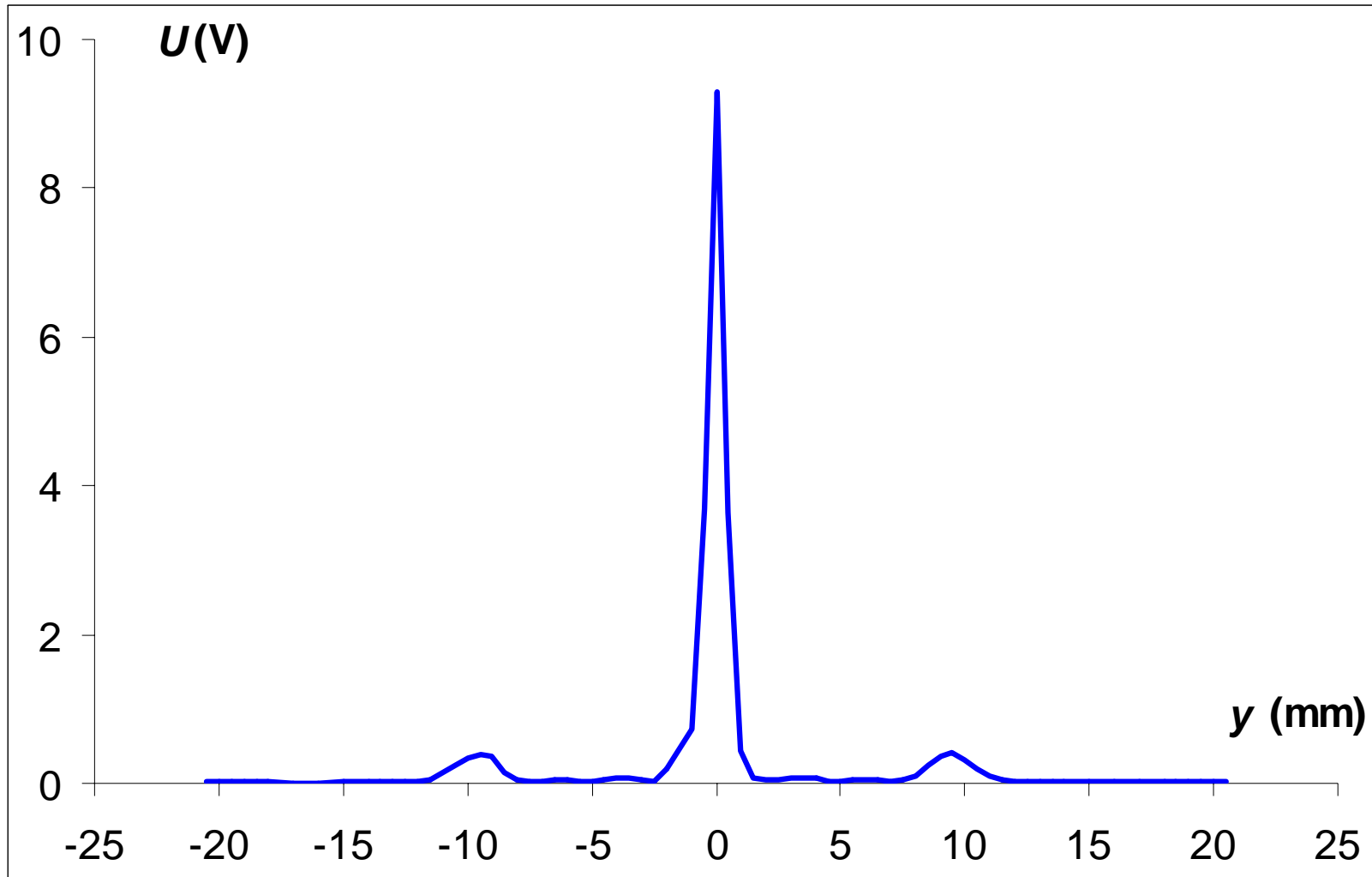
The experimental setup



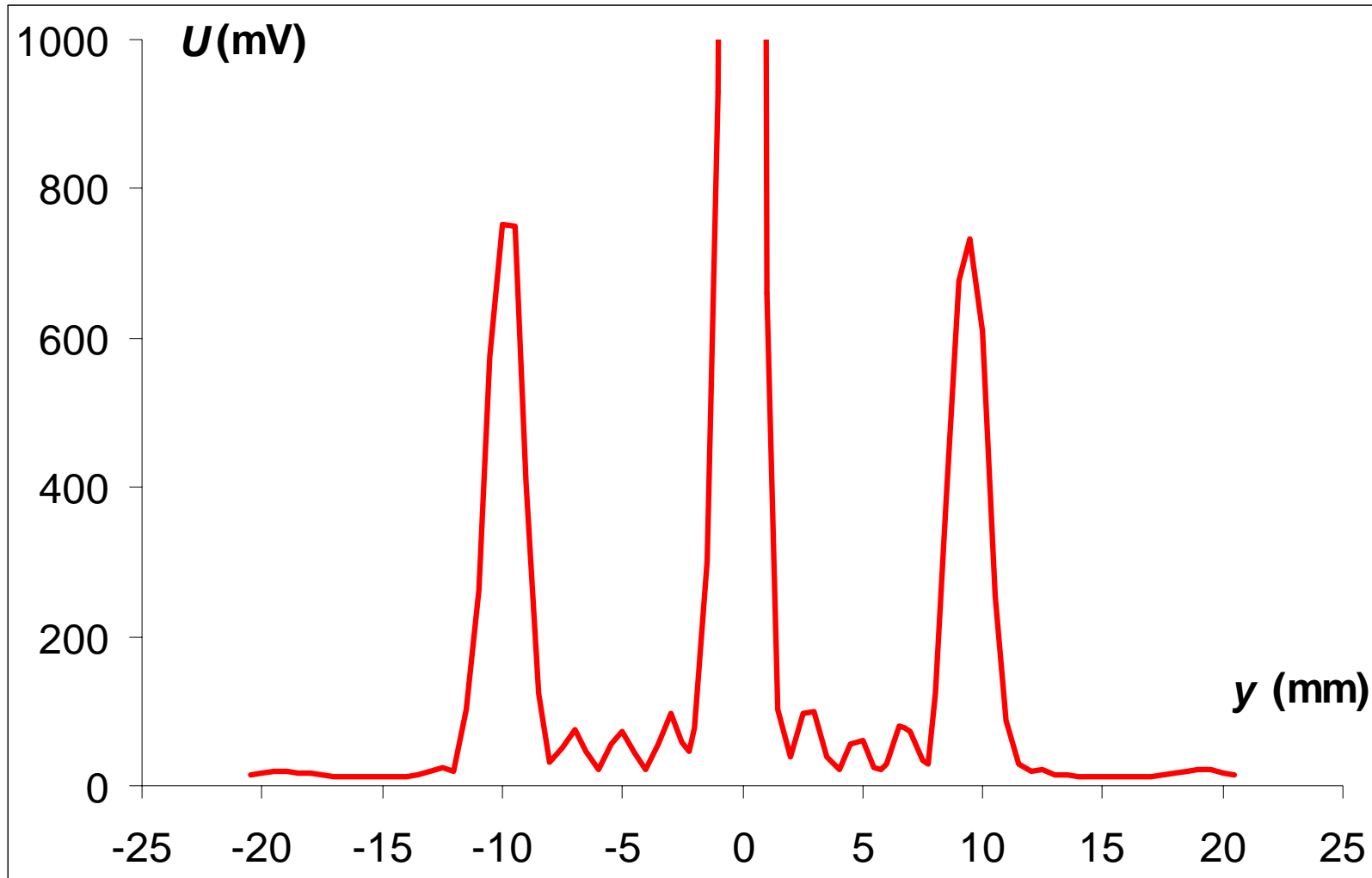
Measured data **A**



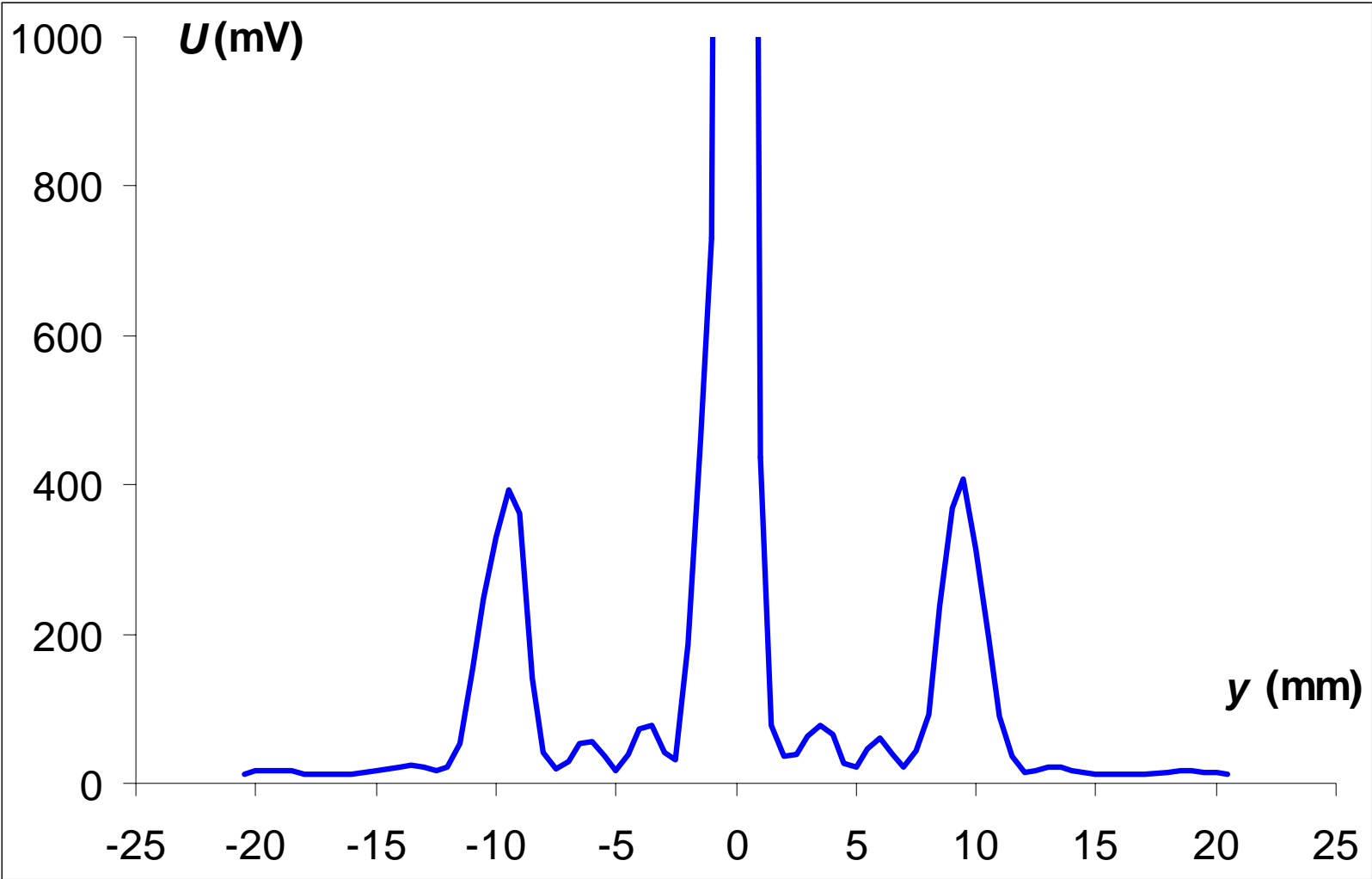
Measured data **B**



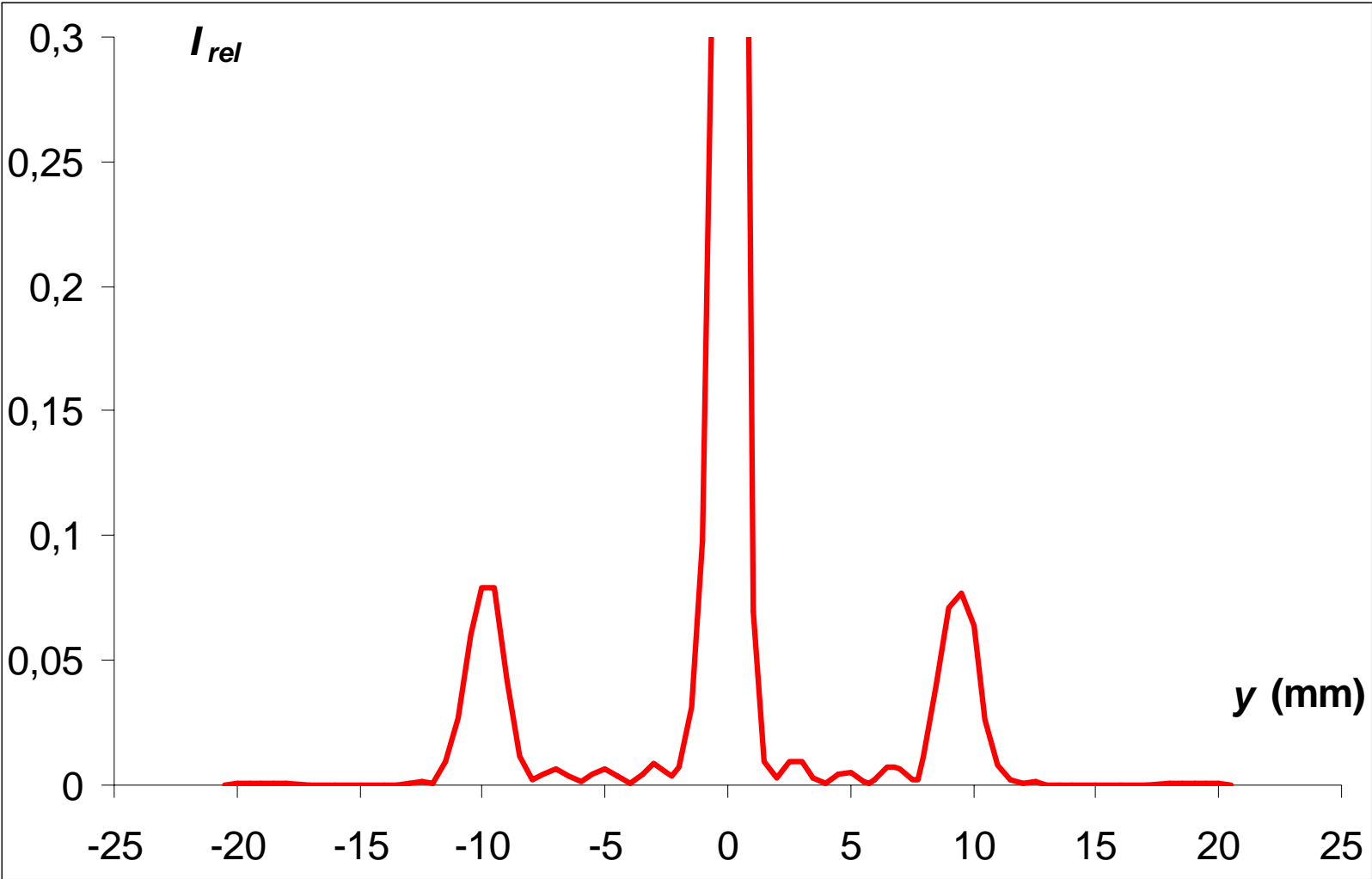
Magnified graph **A**



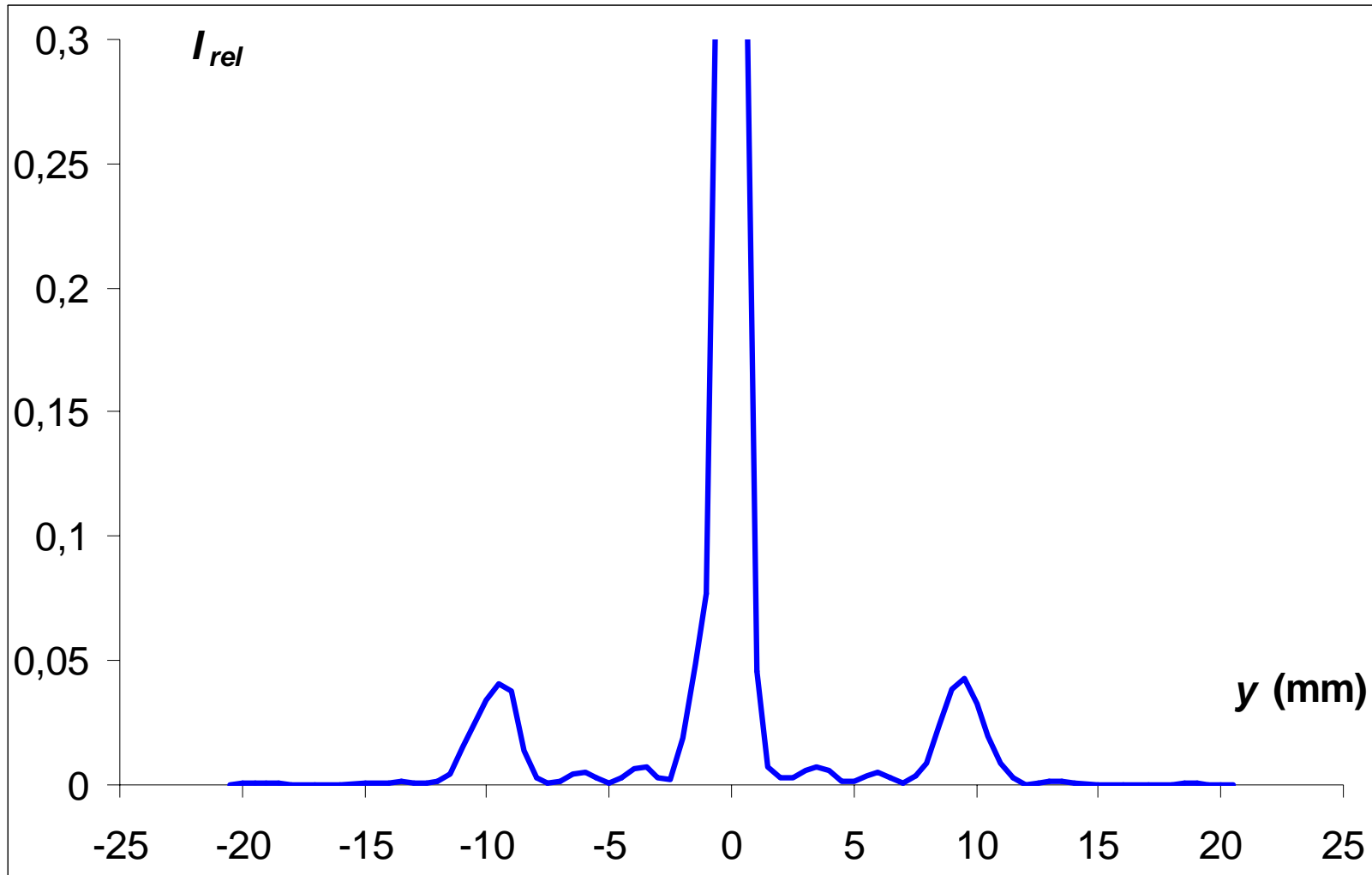
Magnified graph B



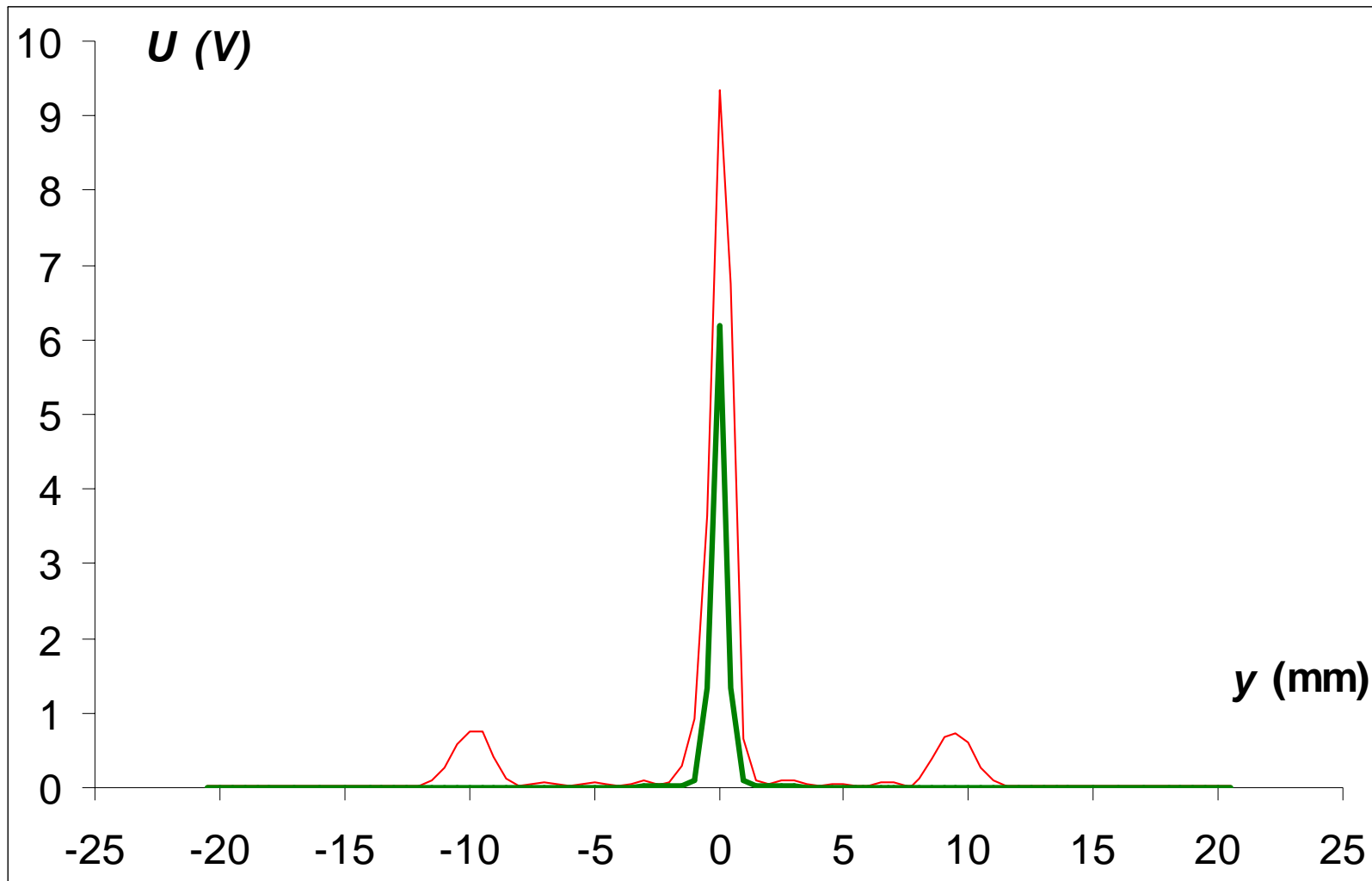
Relative intensity **A**



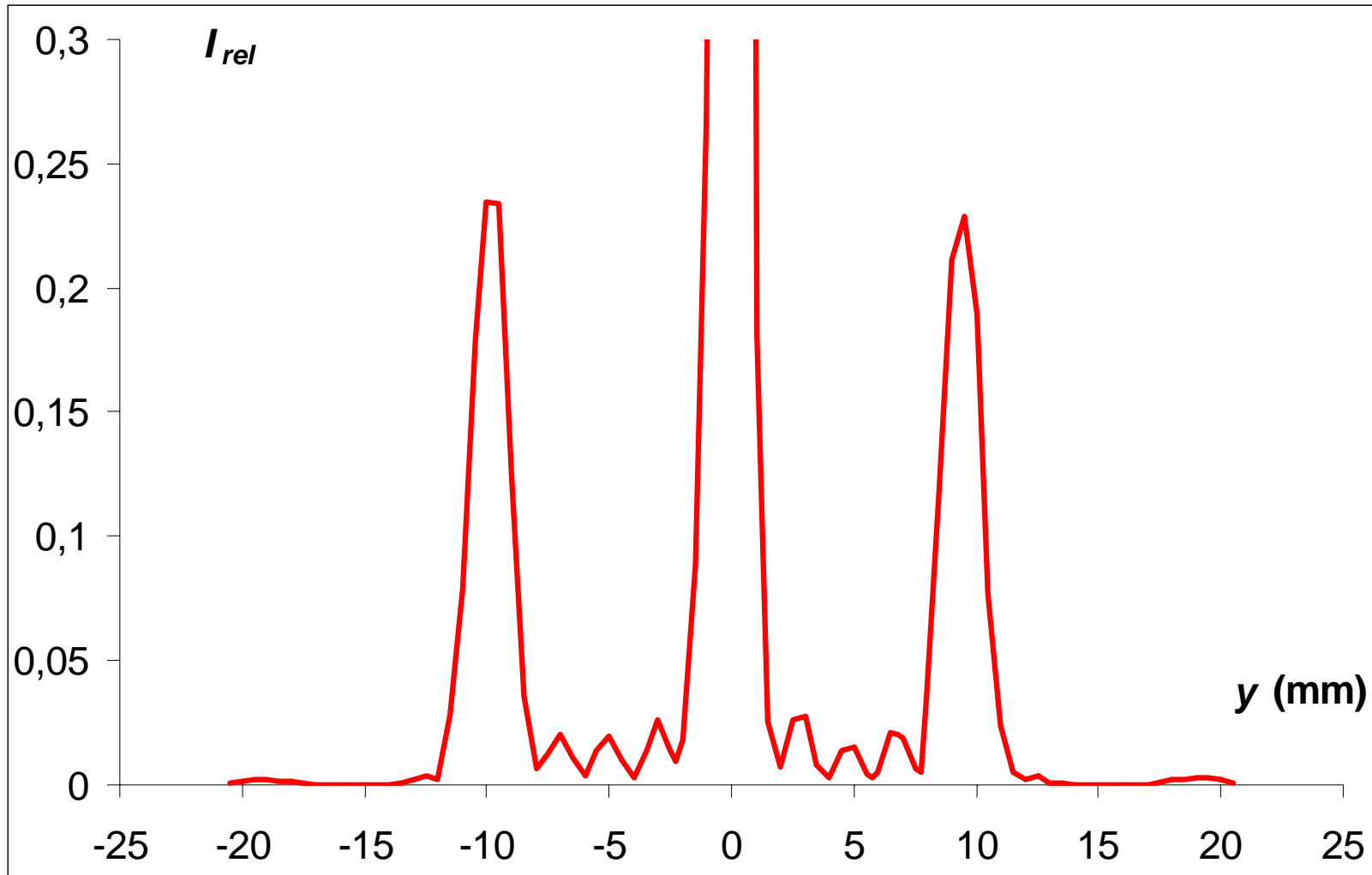
Relative intensity **B**



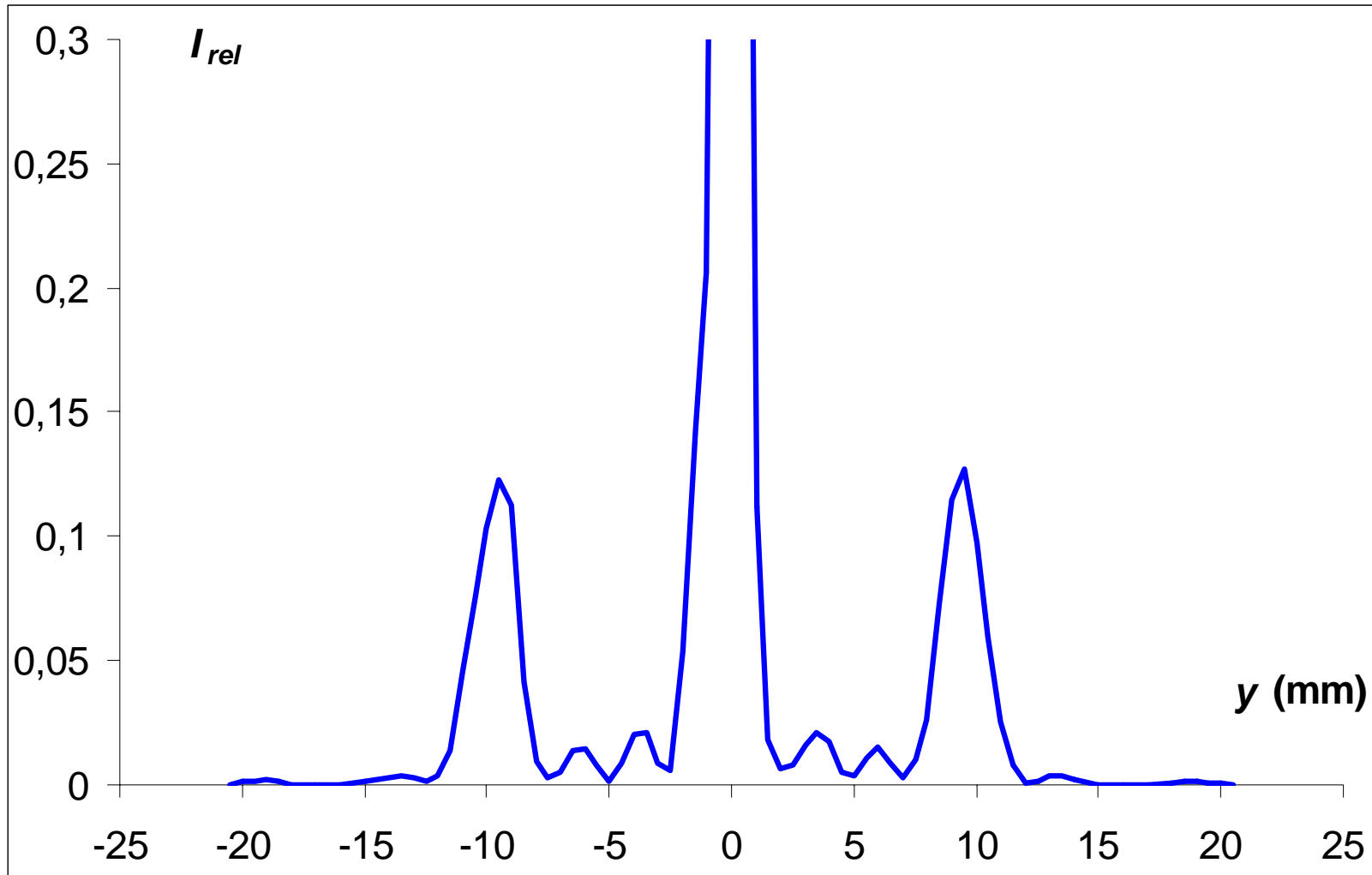
The light intensity across the black background of the slit structures



Corrected graph A



Corrected graph B



Interpretation of the measured data

The **distance** of slits?

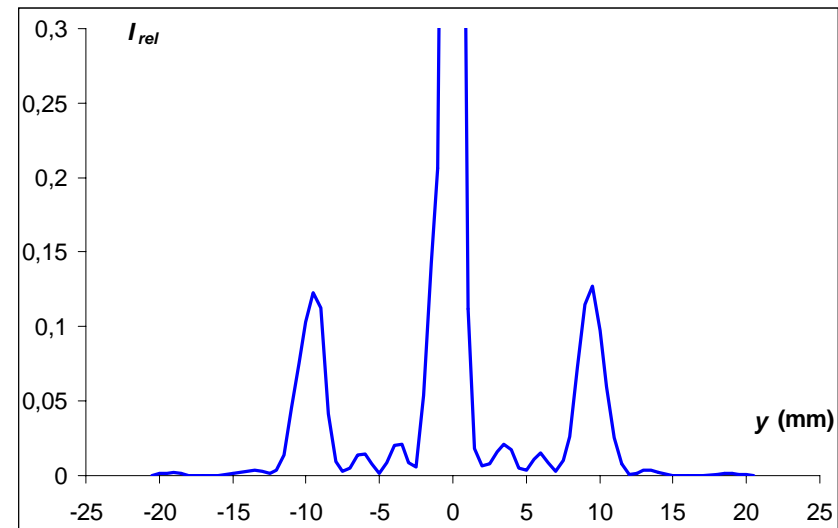
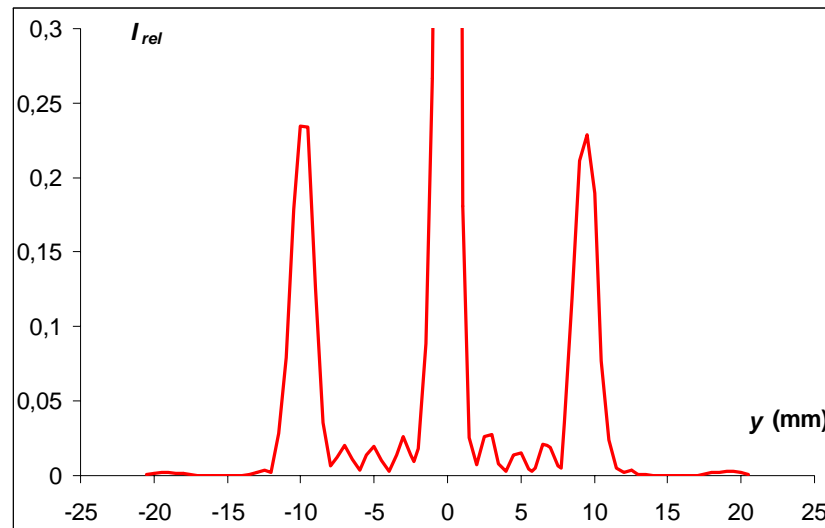
(***d***)

The **number** of slits?

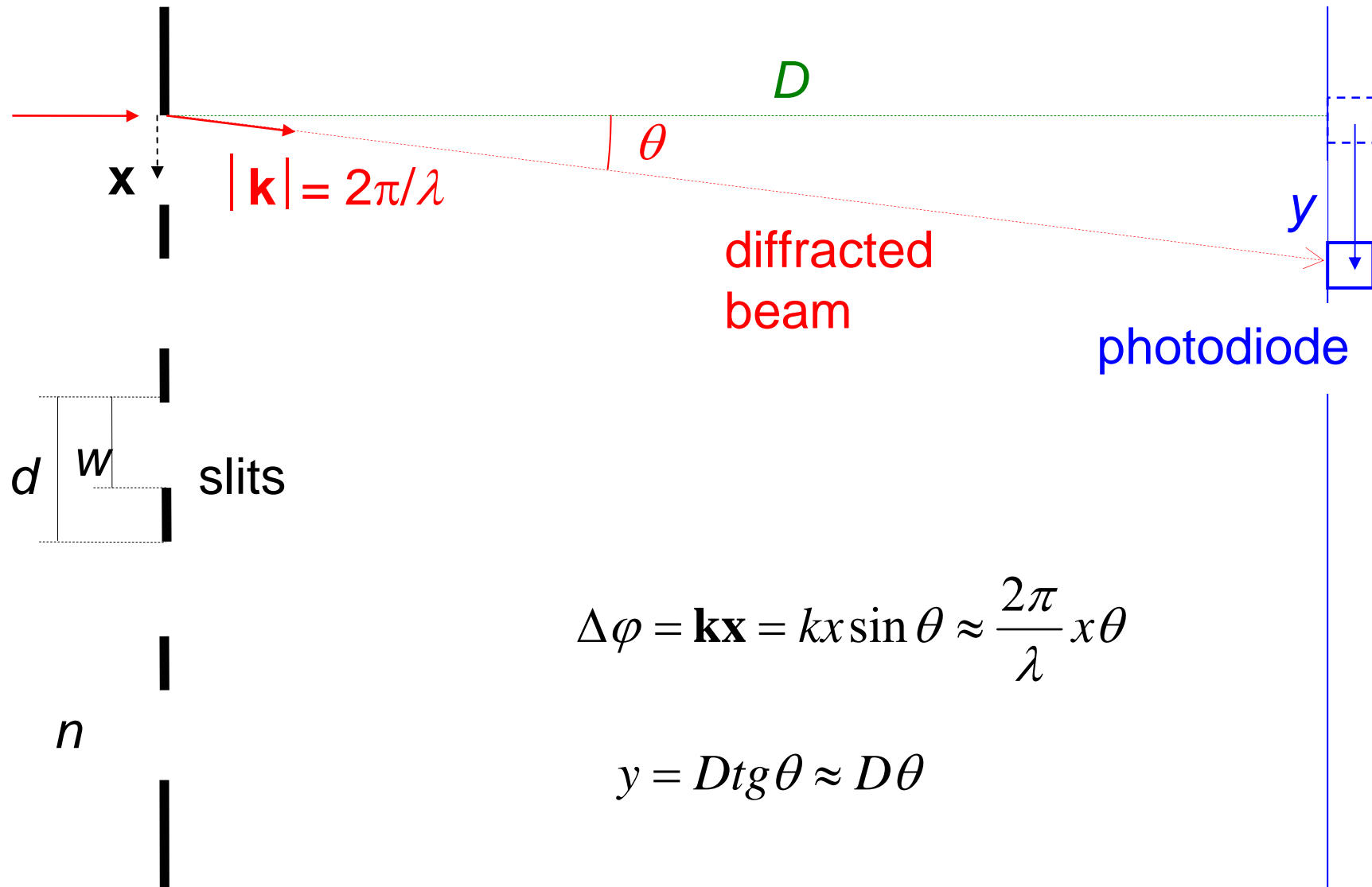
(***n***)

The **relative width** of slits?

(***w/d***)



Diffraction on the multiple slit



Determination of d

The position of the first maximum:

$$\frac{\lambda}{d} = \sin \theta \approx \theta \qquad y = D \tan \theta \approx D \theta \approx D \frac{\lambda}{d}$$

(as for the double slit or for the grating)

$$y = 9.75 \pm 0.25 \text{ mm} \quad D = 1 \pm 0.005 \text{ m} \quad \lambda = 650 \pm 7 \text{ nm}$$

$$\mathbf{d = 67 \pm 3 \text{ } \mu\text{m for both structures}}$$

Determination of n

The **number of slits** is related to the **number of small maxima** (or to the **number of zeros**) between two neighboring big maxima.

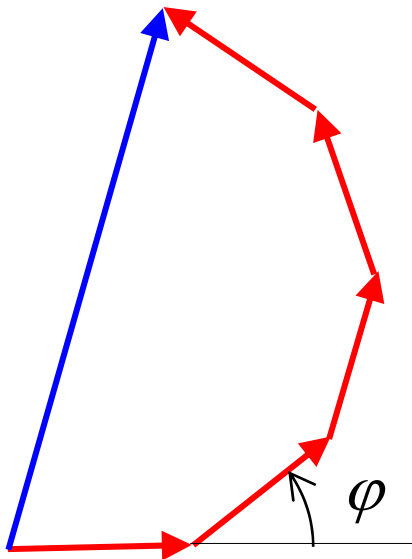
To understand the relationship: **use phasors**.

Phasors are (rotating) vectors expressing **phase** and amplitude of a quantity.

Phasor representation of the E vector of the light

The phase difference between neighboring slits is

$$\varphi = \frac{2\pi}{\lambda} d \theta$$

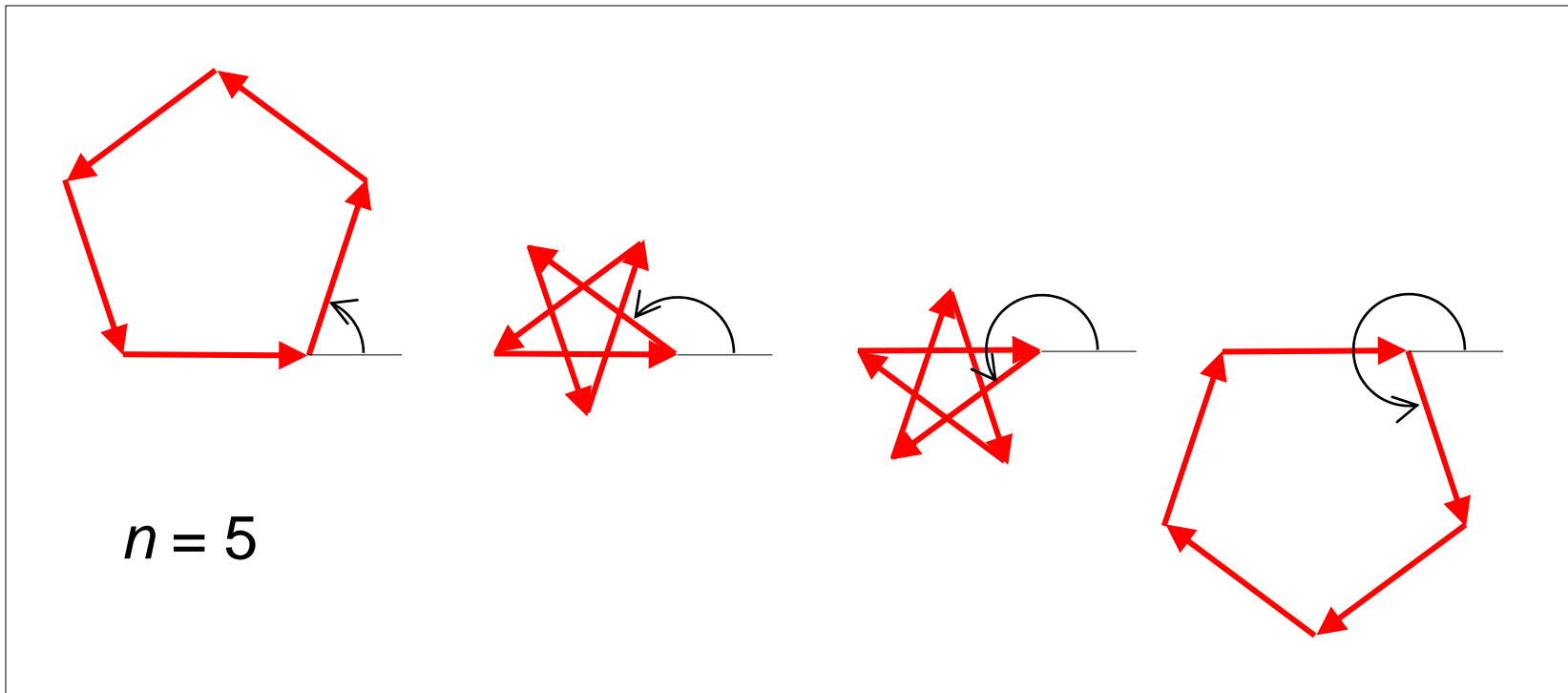


The light intensity is proportional to the square of the (vectorial) **sum** of the **phasors**.

Zeros

The sum of n identical phasors can be zero if

$$\varphi = m \frac{2\pi}{n} \quad m < n \text{ is integer}$$

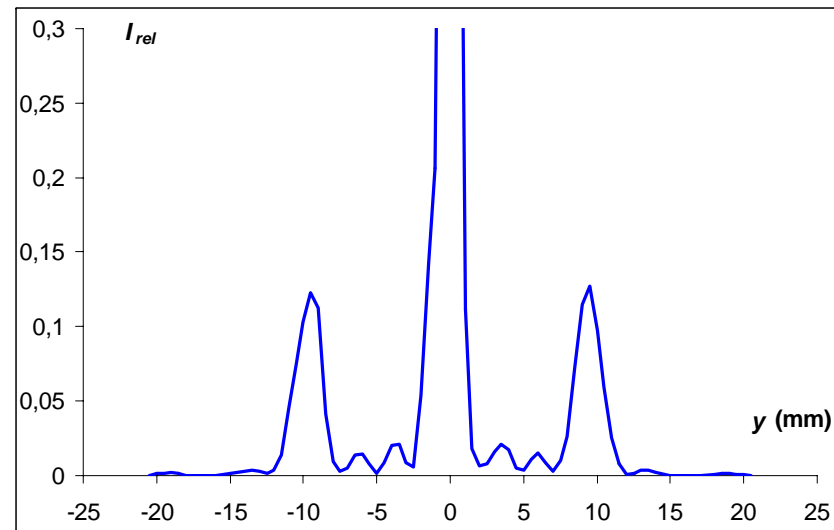
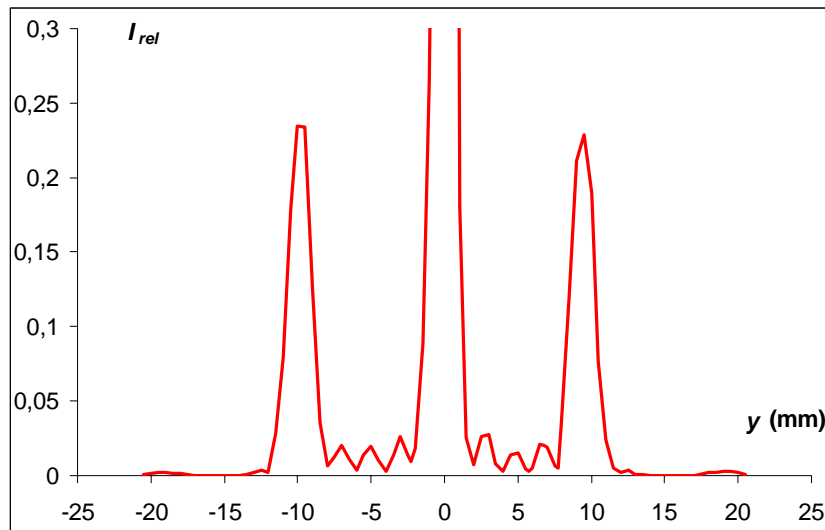


Determination of n

$n-1$ zeros and $n-2$ small maxima
between two big maxima

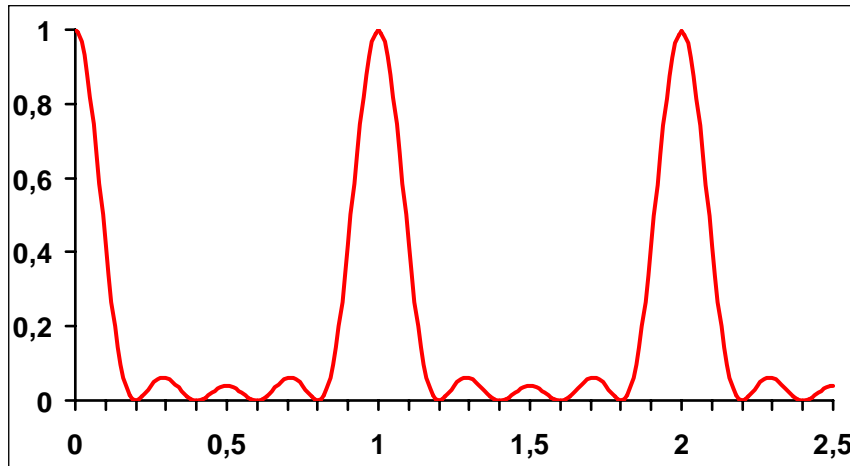
$n = 5$ for structure A

$n = 4$ for structure B

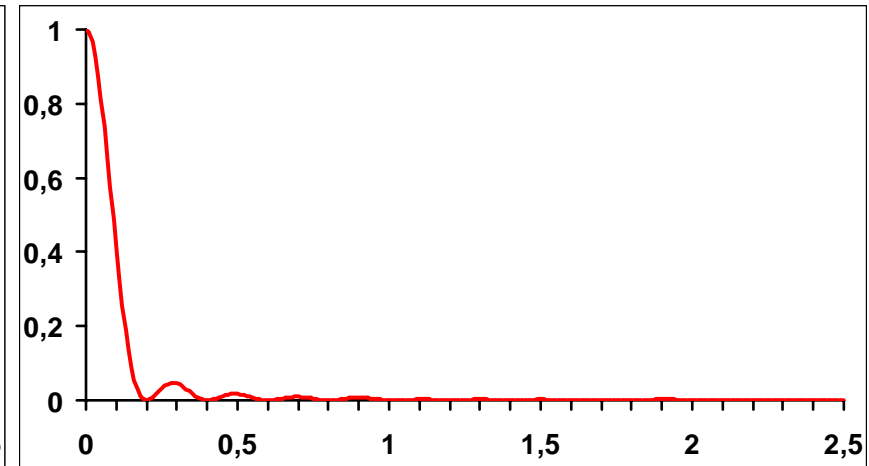


Determination of w/d

Neither $w/d \approx 0$ nor $w/d \approx 1$ is possible.



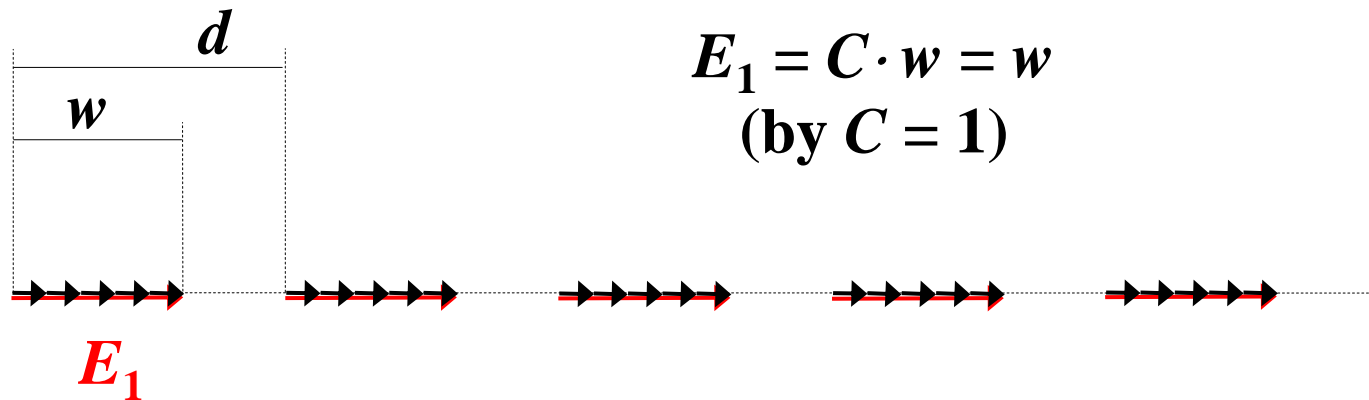
The maxima would have the same (similar) intensity.



It would behave as a single slit with $d' = nd$.

Phasor diagram of the multiple slit

$$n = 5 \quad w/d = 5/8 \quad \theta = 0$$

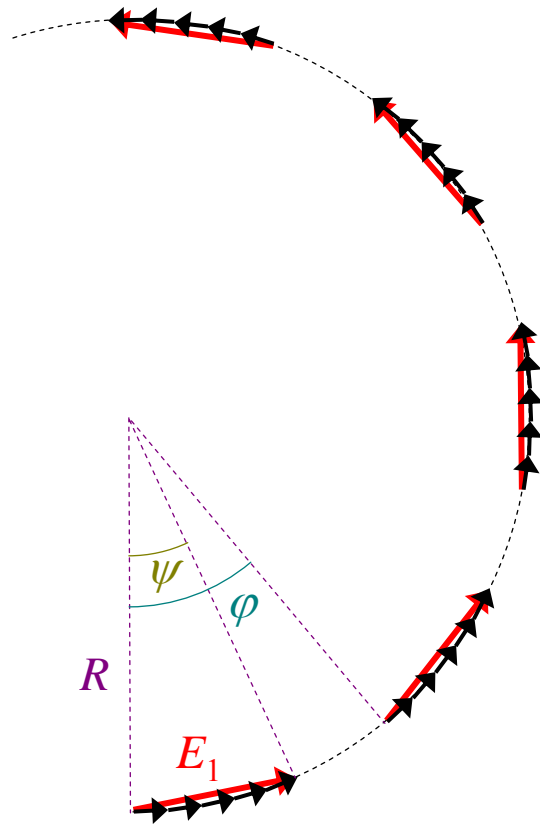


The resultant electric field E_1 is shown as a single long red arrow pointing to the right, representing the constructive interference of the fields from all slits.

$$I_0 = |\vec{E}|^2 = n^2 w^2$$

Phasor diagram of the multiple slit

$$n = 5 \quad w/d = 5/8 \quad \theta > 0$$



$$\frac{1}{R} = \frac{2\pi}{\lambda} \cdot \theta$$

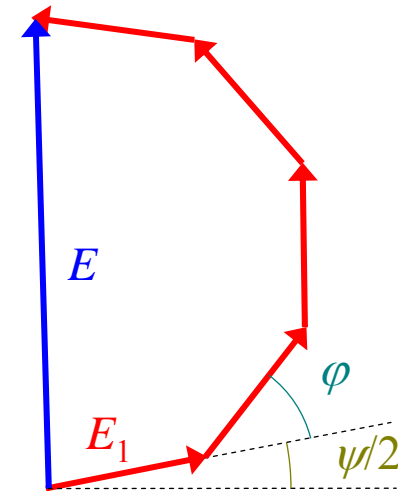
$$\varphi = \frac{d}{R} = \frac{2\pi}{\lambda} \cdot \theta \cdot d$$

$$\psi = \frac{w}{R} = \frac{2\pi}{\lambda} \cdot \theta \cdot w$$

$$E_1 = 2R \sin\left(\frac{\psi}{2}\right) = \frac{\lambda}{\pi\theta} \sin\left(\frac{\pi\theta}{\lambda} w\right)$$

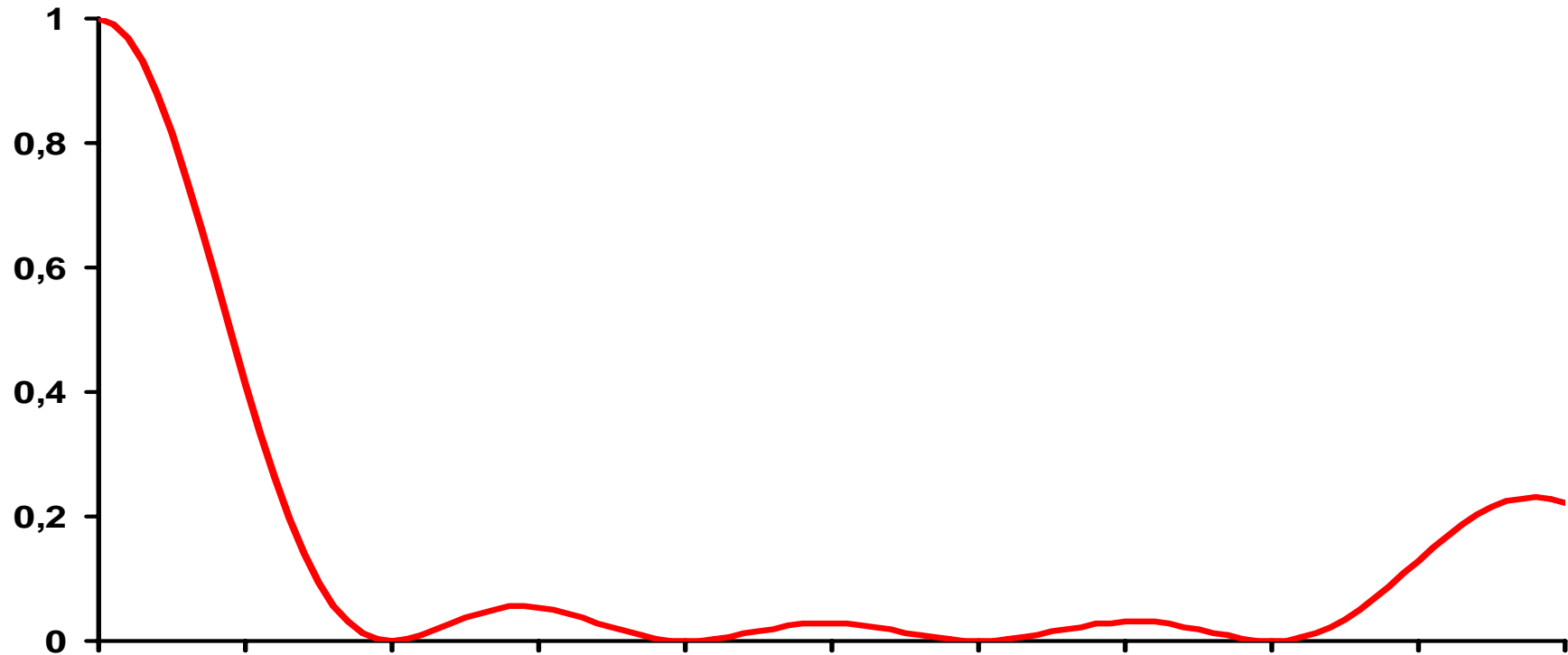
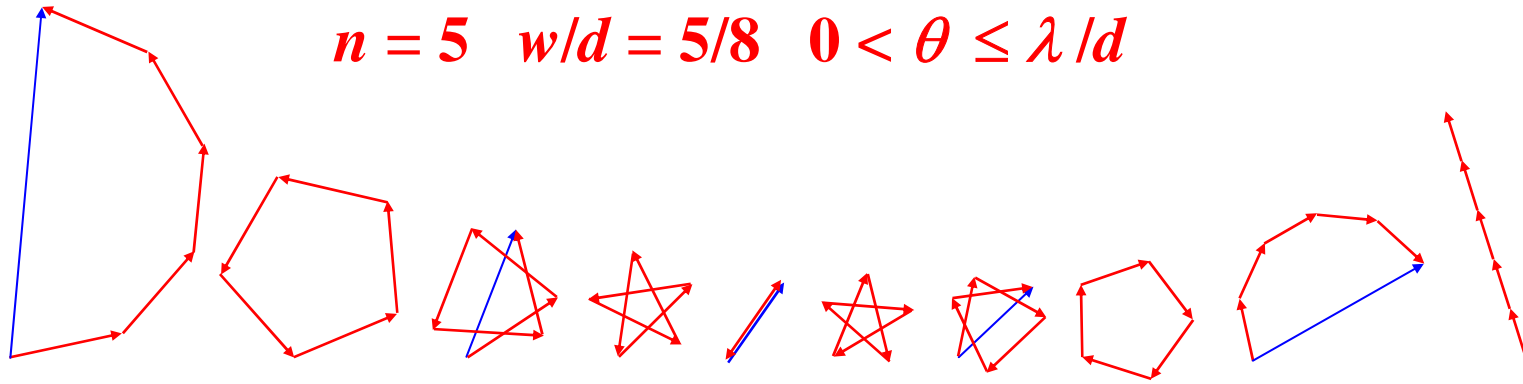
$$\vec{E} = \sum \vec{E}_i$$

$$I = |\vec{E}|^2$$



Phasor diagram of the multiple slit

$$n = 5 \quad w/d = 5/8 \quad 0 < \theta \leq \lambda/d$$



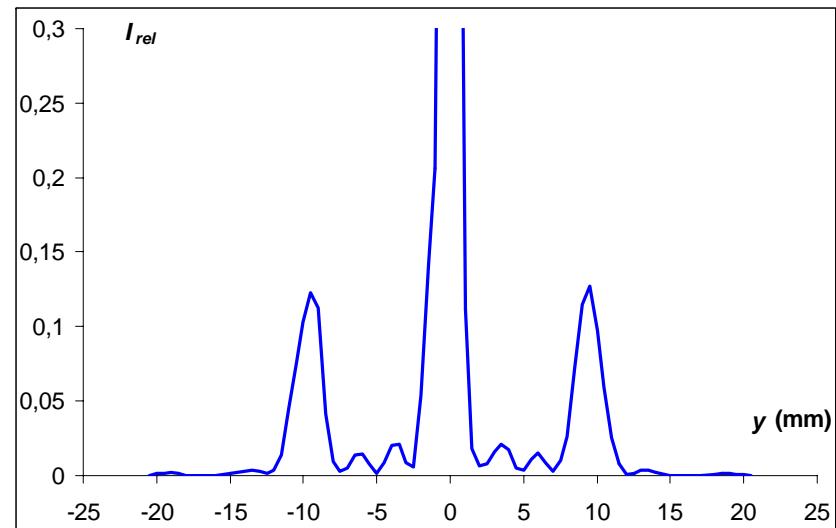
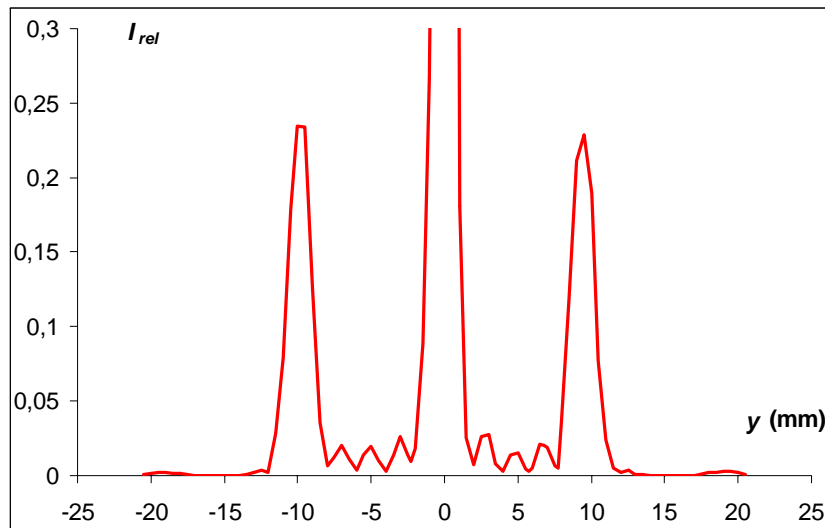
Determination of w/d

The intensity of the first big maximum:

$$I_{rel} = \frac{I}{I_0} = \left(\frac{d}{w\pi} \right)^2 \sin^2 \left(\frac{w\pi}{d} \right)$$

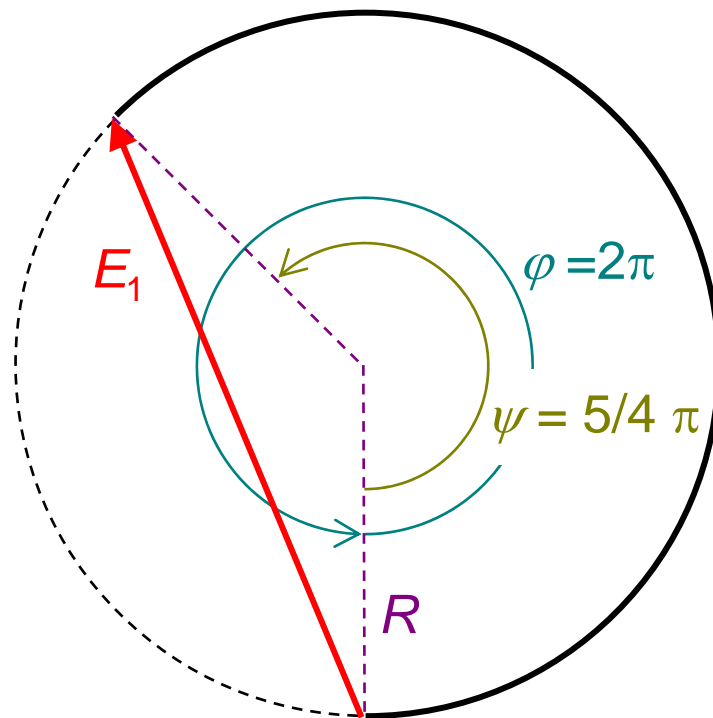
$$I_{rel} \approx 0.25 \Rightarrow w/d \approx 0.6$$

$$I_{rel} \approx 0.15 \Rightarrow w/d \approx 0.7$$

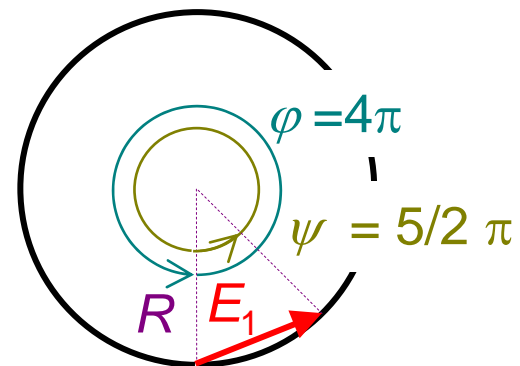


Determination of w/d

Investigation of the second big maximum:

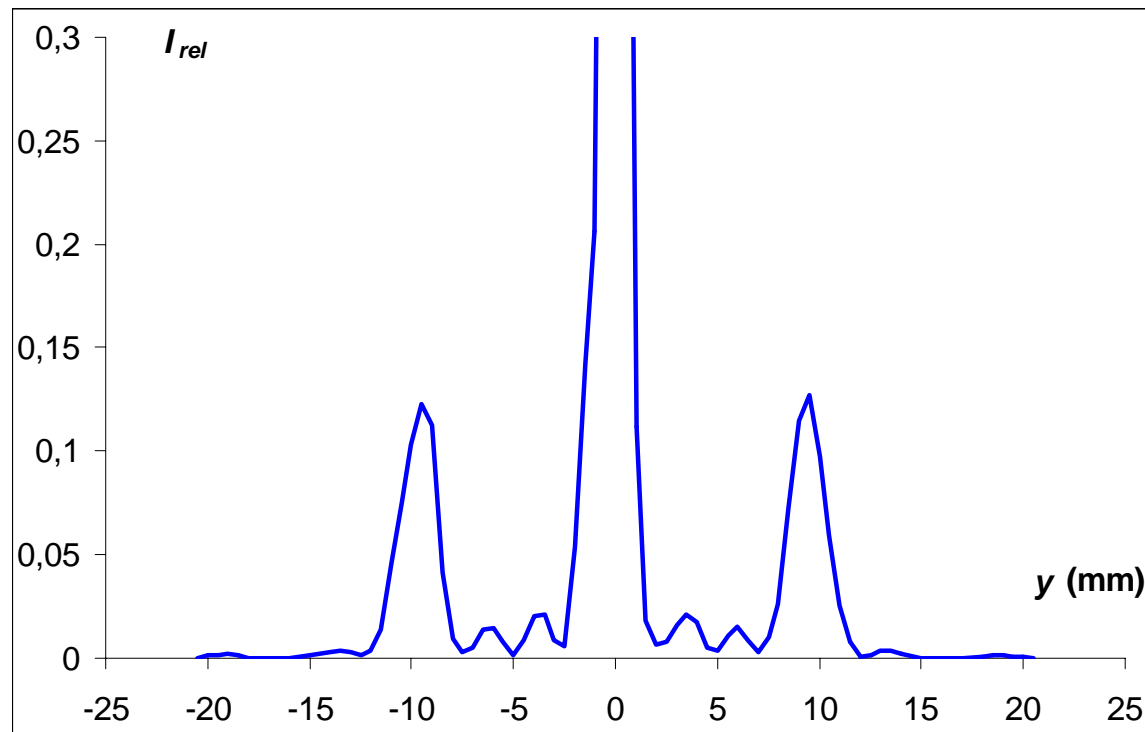


$$w/d = 0.625$$



Determination of w/d

Investigation of the second big maximum:



$w/d \approx 0.5$

Determination of w/d

Considered both argumentations

$$w/d \approx 0.55 \pm 0.05$$

The concept of the workshop

- An afternoon event - **free** from the syllabus
- For inquiring **students** and **teachers**
- The participation is **voluntary**
- **Interdisciplinary** approach
- **Enough time**
 - for mathematical and physical **background**
 - for nice **experiments** and **measurements**
 - to analyze measured data by **PC software's**
 - to **discuss** details and the consequences

Mathematical background

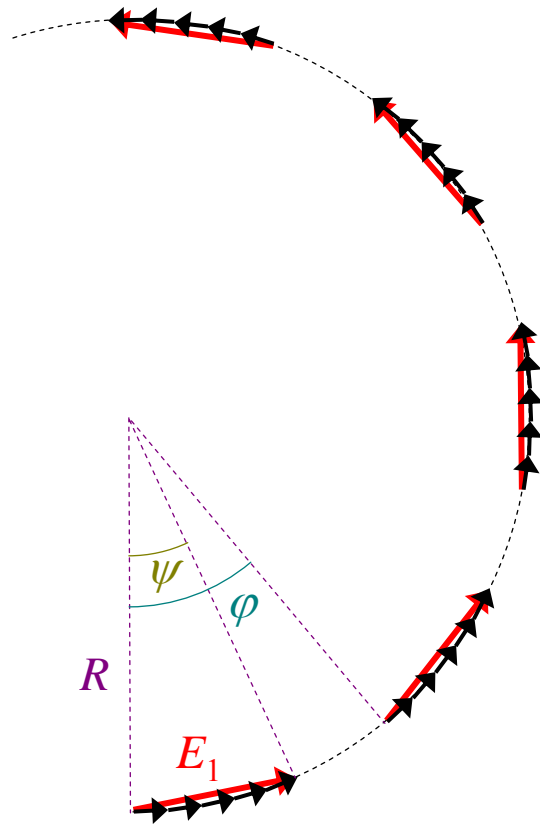
Fourier-series and **Fourier-integral**
(introduction without exactness)

Diffraction of light on the multiple slit as a
physical realization

$$I(\mathbf{k}) = \left| \int_{-\infty}^{\infty} \exp(i\mathbf{k}\cdot\mathbf{x}) f(\mathbf{x}) d\mathbf{x} \right|^2$$

Phasor diagram of the multiple slit

$$n = 5 \quad w/d = 5/8 \quad \theta > 0$$



$$\frac{1}{R} = \frac{2\pi}{\lambda} \cdot \theta$$

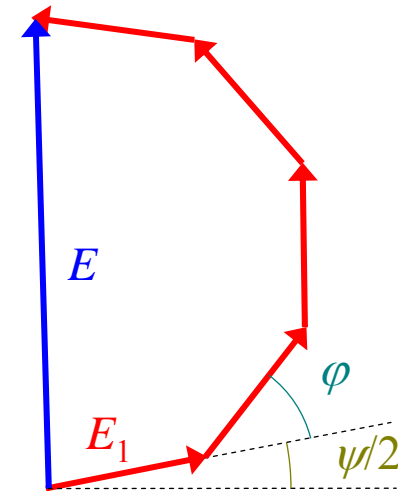
$$\varphi = \frac{d}{R} = \frac{2\pi}{\lambda} \cdot \theta \cdot d$$

$$\psi = \frac{w}{R} = \frac{2\pi}{\lambda} \cdot \theta \cdot w$$

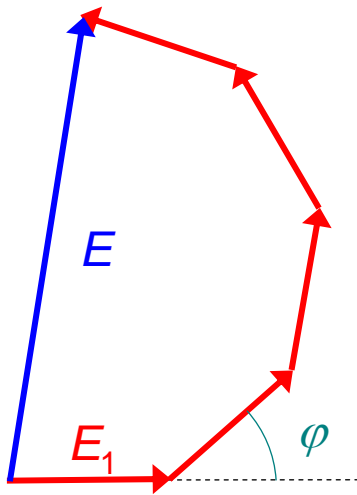
$$E_1 = 2R \sin\left(\frac{\psi}{2}\right) = \frac{\lambda}{\pi\theta} \sin\left(\frac{\pi\theta}{\lambda} w\right)$$

$$\vec{E} = \sum \vec{E}_i$$

$$I = |\vec{E}|^2$$



The calculation of $I_{rel}(y)$



$$E_1 = 2R \sin\left(\frac{\psi}{2}\right) = \frac{\lambda}{\pi\theta} \sin\left(\frac{\pi\theta}{\lambda} w\right)$$

$$\vec{E} = \sum \vec{E}_i$$

$$\varphi = \frac{2\pi}{\lambda} \theta d$$

$$I = |\vec{E}|^2 = E_1^2 \left[(1 + \cos \varphi + \cos 2\varphi + \dots + \cos(n-1)\varphi)^2 + (\sin \varphi + \sin 2\varphi + \dots + \sin(n-1)\varphi)^2 \right]$$

$$I_{rel} = \frac{I}{I_0} = \frac{I}{n^2 w^2}$$

$$y = D \tan \theta \approx D\theta$$

Microsoft Excel worksheet

Conclusions

Playing with the apparatus and the simulation:
a better **understanding** of diffraction

The apparatus, measured data and simulations
can be **used for physics lessons**

The most important effect:
the free atmosphere and the
interdisciplinary approach of the workshop
can **arouse some participants' interest in physics**

Thank You for Your attention

Downloads:

The measured data (Excel)

The simulation (Excel)

The presentation (PowerPoint)

<http://goliat.eik.bme.hu/~vanko/wfphc/wfphc.htm>